ELECTRONIC MATHEMATICS TEXTBOOKS: OLD WINE IN NEW SKINS?

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Mathematics textbooks play different roles in different countries. In some countries they are essential to the quality of instruction and in others they are viewed as not being necessary for experienced teachers. Moreover, the media used to create mathematical texts has advanced from clay tablets to digital ink but this change in the way texts are packaged is not always paralleled in the content of the text. What are some of the educational design features that could help to align the medium of presentation with the content of electronic mathematics textbooks?

When it comes to the current generation of digital mathematics textbooks it is true as Shakespeare noted: "All that glisters is not gold". Many digital mathematics textbooks are simple adaptations of paper texts with the quality of the digital text dependent upon the printed text. Yet digital texts can provide different affordances and constraints to learning mathematics.

WHAT IS THE INTENDED USE OF MATHEMATICS TEXTBOOKS?

In a recent newspaper article (Fiji Sun, January 22, 2011) teachers were asked to "be innovative in their work and not to rely a lot on textbooks". Had this statement from the Minister of Education been made in a different country, say Korea, it would have been met with total disbelief. For many countries, the mathematics textbook is considered to be the most important teaching material (Bae, Sihn, Park, & Park, 2008). In Japan, centrally approved mathematics textbooks are an integral part of the implemented curriculum. Indeed, the School Education Law in Japan states that the use of textbooks is compulsory (MEXT, 2010). Clearly, the use of mathematics textbooks, particularly in primary schools, is looked at differently in different countries. In Australia, the view of progressive elementary mathematics educators is that it is desirable to ween teachers off a dependence on textbooks. This appears to be due, at least in part, to an over-reliance on mathematics textbooks for content knowledge by less confident teachers (Stipek, Givven, Salmon, & MacGyvers, 2001). Where a free market approach is used with the development of mathematics textbooks, that is, *let the buyer beware,* allowing a textbook to become the de facto curriculum is at best a gamble.

Throughout history, mathematics textbooks have been synonymous with mathematics education (Gray, 1990; Love & Pimm, 1996). However, mathematics textbooks in Japan and Australia appear to serve different functions. Many commonly used textbooks in Australia are focused on skill acquisition through a heavy emphasis on procedural practice. This emphasis means that mathematics textbooks are typically voluminous as they contain exercises designed to provide many opportunities for practice, and presumably to allow the teacher to differentiate instruction by allocating different exercises to different students. Rather than having many questions of the same type, Japanese textbooks appear to strive to have fewer questions and focused more on concept development. In exploring differences in mathematics texts it is important to recognise that the cultural practices surrounding the pedagogical use of textbooks shape their content. The exercises used in training scribes in ancient times may not achieve today's goals of educating the total population of countries.

FROM STATIC TEXT TO DYNAMIC TEXT

There is an old parable that states that no one pours new wine into old wineskins. For if he does, the new wine will burst the skins, the wine will run out and the wineskins will be ruined. Traditionally, old wineskins lost their elasticity as they expanded during the fermentation of the new wine. The wineskin was an early device for holding wine, as was the amphora. These containers were in time superseded by the glass bottle. That is, the media used to hold wine has developed over time as have the methods used to control fermentation.

In a similar way, mathematics education has a long history of using different media to communicate ideas. The media used to present mathematics can be thought of as types of *screens*. Initially these screens have been passive: lines drawn in the dust, text on paper, or images projected on physical screens. Almost 4000 years ago the Babylonians used clay tablets to act as screens in recording their mathematics texts (Figure 1).

Figure 1. Yale Babylonian Collection YBC 7289

Yet irrespective of the form of the passive screen, texts do not speak for themselves. For example, the tablet from the Yale Babylonian collection in Figure 1 has been described as "illustrating Pythagoras' Theorem and the square root of 2". However, this is clearly only an interpretation of what the author intended to record. YBC 7289 appears to show a square with the diagonals marked and cuneiform base 60 numerals. To give meaning to the markings on a clay tablet we activate 'mental screens' (Mason, 1992). That is, the images evoked by the markings on the clay tablet can energise our mental screens as we seek to interpret, and sometimes elaborate, a static screen. This process begins by assigning meaning to symbols.

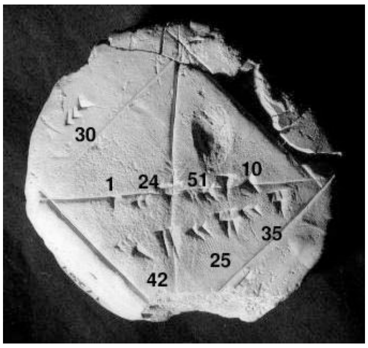


Figure 2. Bill Castleman's annotations to YBC 7289
(http://www.math.ubc.ca/~cass/Euclid/ybc/ybc.html)

The cuneiform figures representing $1+\frac{24}{60}+\frac{51}{60^2}+\frac{10}{60^3}$ (Figure 2) are taken as an approximation of root 2 (an interpretation triggered by the diagonals of a square) with the remaining figures showing what we take to be the calculated diagonal length of a square with side length 30 units. This is quite a lot of meaning that we give to a small number of clay markings. If traditional printed text is open in permitting students to construe what they see in a variety of ways, mathematical images are even more open to ambiguity and to alternative stressing and ignoring. For example in Figure 3, some students interpret only a square in an upright position as an example of a square (Hershkowitz, 1990).

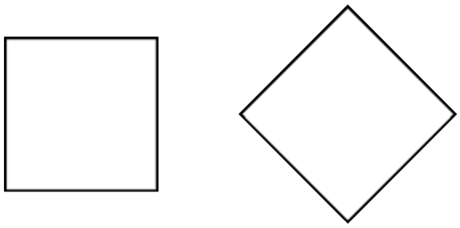


Figure 3. Only one image perceived as a square

Consequently, even the way the tablet is oriented will influence how students interpret what they see. In digital texts as in historical mathematics texts, what we read into a text is always a function of our current knowledge and ways of thinking. The tablet YBC 7289 does not, in my opinion, demonstrate Pythagoras' Theorem in any general sense. It is an example that appears to draw on a good approximation to the length of the hypotenuse of a right isosceles triangle.

The tablet YBC 7289 may also be considered to be an *icon*[1] in two different ways, which can be described as surface and deep observations. Thus, a surface observation of this icon can lead us to think that it may be a representation of a mathematical object. However, in order to move from this idea to seeing it as a figure, as referring directly to the mathematical object itself, requires interpretation.

[1] The term *icon* is used here in the sense of Peirce's semiotic, i.e. as having qualities that resemble those of the object it represents.

Understanding how we use text and images to learn and think about mathematics is necessary in appreciating how electronic mathematics texts can aid student learning. Moreover, the distinction between students *engaging in* mathematical activity or "doing" and students *learning from* activity, which Mason calls "construing" (1992, p. 2) provides a way of seeing the development of mathematics texts as a progression from activities for scribes, to tool-augmented opportunities for learning from activity.

From clay to paper

Moving from clay to paper we have an example of a different text that appears to represent a proof of a specific case of the Gougu (the hook and the leg) Theorem of China, also construed by some as Pythagoras' theorem.

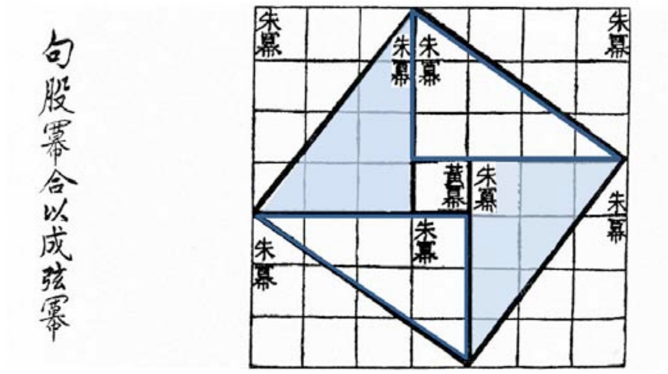


Figure 4. Diagram added to the *Zhou Bi* (周髀) by the 3rd Century commentator Zhao Shuang

Here the image (Figure 4) might be used to give a graphical dissection proof of the gougu relation, although this does not appear to have been the author's intent (Cullen, 2002). Interpreting mathematical traditions can be subject to a form of historical cultural imperialism. Ancient China developed its own mathematical culture based on a radically different approach to the Euclidean structuring of mathematical thought. Indeed, ancient Chinese mathematicians did not talk about right-angled triangles, because they did not talk about triangles in any general sense. Rather than a Euclidean axiomatic approach to mathematics, ancient Chinese mathematicians sought to distinguish categories of problems in order to unite categories.

Figure 4 brings to mind the method of 'piling up of squares' or dissection and rearrangement. In turn, the method of dissection and rearrangement suggests movement and may be better supported by dynamic rather than static mathematics texts. This prompts the question: Is animation an important feature of the new digital mathematics texts?

Watching or doing?

Watch the following animation.

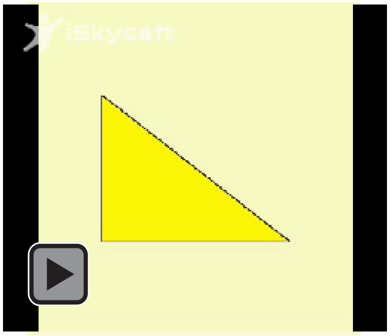


Figure 5. A featured animation on Wikimedia Commons, considered one of its finest images (http://commons.wikimedia.org/wiki/File:Pythagoras-2a.gif)

By watching this animation do you come to understand how to prove Pythagoras' theorem? If not, what else might you need to do? The flow of an animation is often too fast to allow you to ask and answer questions about the mathematics. This brings me to my first design challenge for electronic mathematics textbooks.

DESIGN CHALLENGE 1. Animation is not a substitute for interaction.

Objects that can be manipulated directly are usually easier to understand and help you to get a feel for the mathematics. If animation is used in a mathematics digital text, it is best to place the controls of the animation in the hands of the user. Many internet websites make use of animations to demonstrate dissection and rearrangement proofs of Pythagoras' Theorem (e.g. http://sunsite.ubc.ca/DigitalMathArchive/Euclid/java/html/pythagorasdissection.html). While these use the power of digital technology to portray dynamic imagery they also bring to the fore the challenge of distinguishing between students *engaging in* mathematical activity or "doing" and *learning from* activity. It is quite easy to play these animations repeatedly without learning from the activity.

Writing mathematics

The technology of printing allowed ideas to be shared and made available to the general population. However, the physical limitations of printing impacted on the adoption of ways of symbolising mathematics and the creation of mathematics texts. For example, in 1659 the Swiss, Johann Heinrich Rahn, published an algebra in which he introduced ÷ as a sign of division. Many writers before him had used ÷ as a minus sign. English translations of Rahn's work led to the eventual popularisation of ÷ for division but this was not a simple process. For some time, both ÷ and : were used as symbols for division. The former predominantly belonged to Great Britain and the United States. The latter belonged to continental Europe and the Latin-American countries.

Leibniz introduced, and to some extent popularised, the colon as the symbol for division in Europe and frequently used a colon when writing fractions, leading to rendering 1:3 = $\frac{1}{3}$. Symbolising as a form of representation influences our appreciation of mathematics and our capacity to do mathematics. We rely on the symbol system to carry some of the cognitive load associated with thinking through problems. Yet typesetting input systems restrict the flexibility of the symbol system used with mathematics. Mathematicians writing papers or creating formal presentations in current times often rely on special typesetting languages, such as LaTex, to write even simple fractions: {$${\frac {1}{3}}$$}. One of my mathematics teachers used to frequently remind me that you always read mathematics with a pencil in your hand. This becomes rather difficult if we change totally from a paper-based medium to a digital medium. However, as we do not currently write in mathematics textbooks this should not be an insurmountable obstacle.

DESIGN CHALLENGE 2.

Recording mathematics should be as natural as possible but need not be a requirement of a mathematics e-text.

Using mathematical storyboards

In teaching mathematics we often seek to establish a general result using a form of inductive reasoning to move from specific cases to a general rule. If we choose our examples carefully we can also build the foundation for deductive reasoning.

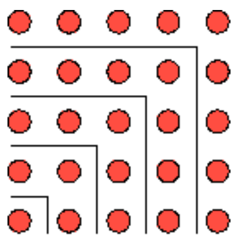


Figure 6. An image showing 5^2 as the sum of the first 5 odd integers

One image, as in Figure 6, or even a series of images doesn't prove a general result to our present expectations of proof[2]. However, this diagram can help us to see that to get from 4^2 to 5^2 we add on (2 x 5 – 1). That is, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$. This result is often one of the first examples used to teach proof by mathematical induction. The value of the arrangement of the specific cases in this example assists

[2] Before the development of symbolic algebra, geometric demonstrations were considered valid modes of proof (see *Proofs Without Words* by R. B. Nelson, MAA, 1993).

us to see how to create the general term. Further, the diagrammatic method can be used to help students arrive at mathematical concepts independently with a sense of why the result must be true. The diagrammatic method can also provide clues to steps in a deductive proof. One of my hopes is that digital mathematics textbooks will provide a vehicle for the evolution of an interactive diagrammatic method.

When we create an animation it is common practice to develop a summary of the action as a storyboard of the key frames. The key frames capture significant developments in the story. Consider the following as frames in a simple storyboard dealing with a specific example of the Pythagorean relationship (Figure 7).

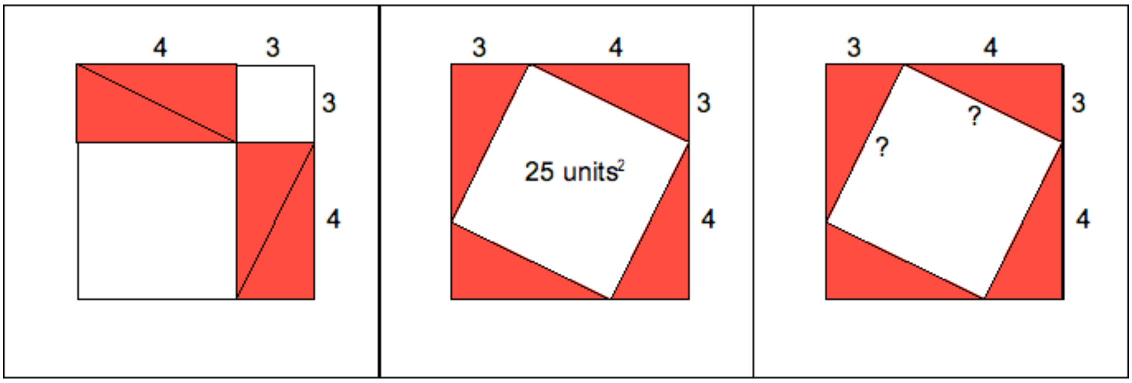


Figure 7. A storyboard to find the length of the bowstring (hypotenuse)

Digital textbooks can provide simple interactivity to enable students to rearrange the geometric shapes in Figure 7. Providing space to record reasoning and calculations below each frame can provide a structure that could support the use of the diagrammatic method. Students can discern the relationships contributing to the desired generalisation (see Figure 8).

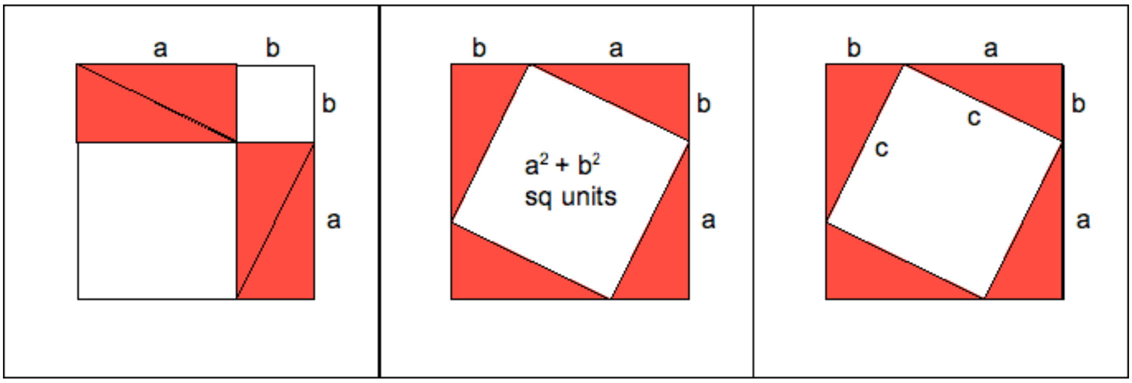


Figure 8. Generalising the values in the storyboard

Storyboards can be used to link written argument with the diagrams. Alternatively blank frames may be used to create opportunities to add in missing steps in the story. The best way to use the diagrammatic method with digital mathematics textbooks is a question that can ultimately be answered empirically. However, the potential of interactive diagrams is important in the design of digital textbooks.

DESIGN CHALLENGE 3.

The interactivity and experimentation possible with computers should enable digital mathematics textbooks to make effective use of the diagrammatic method.

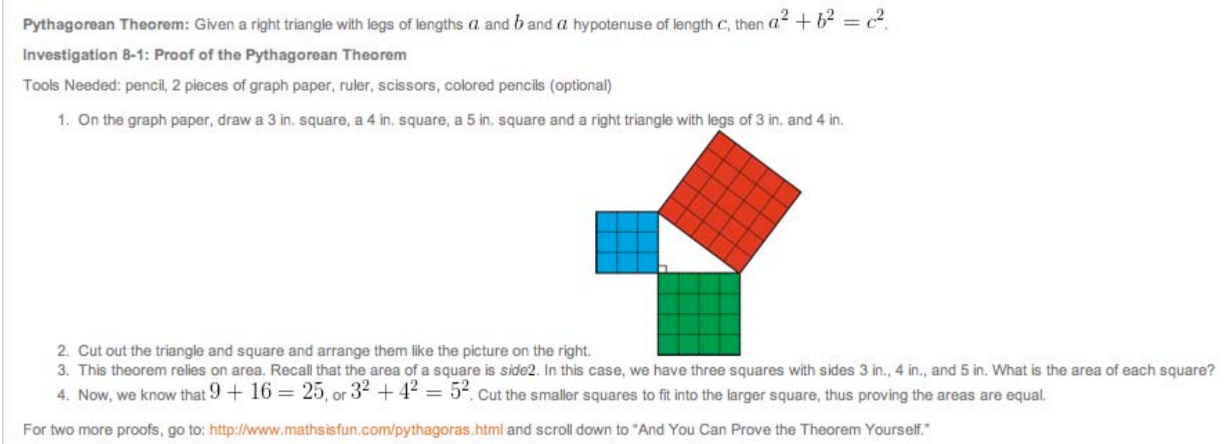
A DIGITAL TEXTBOOK PROGRAM

"All that glisters is not gold."

The Merchant of Venice (II, vii)

Although the limitations of traditional paper textbooks are well known in terms of their expense, weight and rigidity of format, the strengths and limitations of using electronic textbooks for mathematics content need to be articulated. Several educational jurisdictions have announced a commitment to digital textbooks. For example, the Education Ministry of South Korea announced the Digital Textbook program on March 8, 2007. The digital textbook is currently being tested in several primary schools in Korea with the intention of distributing it free to every school nation-wide by 2013.

On a different scale, California announced a Free Digital Textbook Initiative in 2009 with a focus on high school textbooks. One source of free digital mathematics textbooks is the CK-12 Foundation, a non-profit organisation attempting to reduce the cost of textbook materials. CK-12 Foundation uses an open-content, web-based model they call the FlexBook. FlexBooks are intended to conform to national and state textbook standards. However, if we look at how Pythagoras' theorem is addressed in FlexBooks, we see that it is very much 'old wine in new skins' (Figure 9).



Pythagorean Theorem: Given a right triangle with legs of lengths a and b and a hypotenuse of length c, then $a^2 + b^2 = c^2$.

Investigation 8-1: Proof of the Pythagorean Theorem

Tools Needed: pencil, 2 pieces of graph paper, ruler, scissors, colored pencils (optional)

1. On the graph paper, draw a 3 in. square, a 4 in. square, a 5 in. square and a right triangle with legs of 3 in. and 4 in.
2. Cut out the triangle and square and arrange them like the picture on the right.
3. This theorem relies on area. Recall that the area of a square is *side*2. In this case, we have three squares with sides 3 in., 4 in., and 5 in. What is the area of each square?
4. Now, we know that $9 + 16 = 25$, or $3^2 + 4^2 = 5^2$. Cut the smaller squares to fit into the larger square, thus proving the areas are equal.

For two more proofs, go to: http://www.mathsisfun.com/pythagoras.html and scroll down to "And You Can Prove the Theorem Yourself."

Figure 9. Traditional resources described in a FlexBook
http://www.ck12.org/flexbook/chapter/8938

The FlexBook approach to the 'proof' of Pythagoras' Theorem ignores the digital medium and describes the 'Tools Needed' as traditional pencil, paper and scissors. The question of what tools are needed with a digital mathematics textbook is important in the development of digital resources.

The Mathematics Digital Textbooks used in the South Korean program also copy the content of printed textbooks. The Digital Textbooks are used on a tablet PC with input from a stylus and the keyboard. In a report on the problems of learning with the Mathematics Digital Textbook for Grade 6 (Choi, J-I, Yum, M., & Lee, Y-T, 2010) one of the challenges identified was *representing* the mathematics in both the original printed textbook and the digital textbook. For example, when dealing with division of fractions ($\frac{5}{6} \div \frac{2}{6}$) the majority of the students in the study obtained an incorrect answer (commonly, 3) using the representation provided in the textbook (Figure 10).

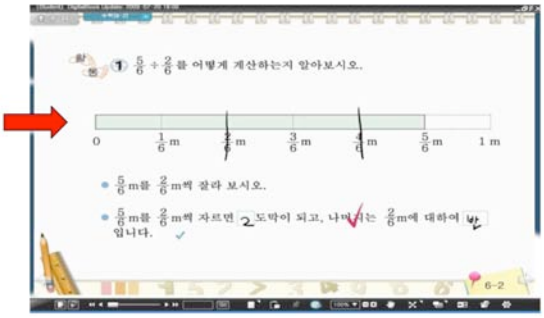


Figure 10. Dividing 5-sixths by 2-sixths using the Korean DigitalBook

When subdividing $\frac{5}{6}$m to determine how many times $\frac{2}{6}$m would divide into it, many students appeared to focus on the number of pieces formed. The introductory animation showing students how to cut a ribbon also divided the ribbon into three pieces, where the size of each piece was not the same.

Another problem identified with the Digital Textbook was the sequencing of content. Examples that drew upon dividing one integer by another frequently elicited decimal responses rather than answers in common fractions, as this was the topic learnt prior to the unit on division of fractions. Also, on some activities it was not obvious how to use the mouse to achieve the task goal. That is, the interactive tool kit did not have a natural interface or visible *affordance*. The term affordance was used initially by the perceptual psychologist J. J. Gibson (1977) to refer to the actionable properties between the world and an actor. Put more simply, a perceived affordance typically describes whether the user perceives that some action is possible. This was one of the problems identified with the task in Figure 11 (Choi, J-I, Yum, M., & Lee, Y-T, 2010).

Figure 11. Students had to double-click to drag two pieces (sixths) at the same time

Information from the evaluation of the Korean Digital Textbook program identifies the need for sound content sequencing as it relates to the development of representational tools. It also prompts pedagogical questions about the use of representations.

DESIGN CHALLENGE 4.
Students should always attempt a new type of mathematics problem with a familiar representational tool.

As learners come to use particular representations in learning activities, the representations help guide the learning process. How representational tools are introduced and how connections are made between the uses of particular tools are important considerations in the development of digital textbooks. Tool use influences the nature of external behaviour and also the mental functioning of individuals (Vygotski, 1978).

USING TOOLS WITH DIGITAL MATHEMATICS TEXTBOOKS

Identifying the types of tools used to support mathematics learning is important in the design of effective digital mathematics texts. If the question of which representational tools need to be developed in a mathematics curriculum has been addressed in the existing mathematics textbooks, the development of a digital textbook is predominantly a technical challenge. Software such as *dbook* (CRICED, 2008) can help in addressing the technical challenge. However, *dbook* starts from the premise that the digital text is based on a high quality paper-based textbook. This means that decisions related to appropriate representational tools have often been made before the digital text is created. In many countries, these decisions on developing appropriate representational tools and how they might be portrayed in texts are yet to be made. Even the awareness that this is an important question in curriculum design and delivery may not be evident.

For example, the tape diagram is an important representational tool used in Japanese mathematics textbooks as well as in those in Singapore (Beckman, 2004; Murata, 2008). Tape diagrams are used for multiple mathematics topics and are developed across several grade levels. Drawings of linear arrangements of objects are used as precursors to the introduction of single tape diagrams to represent addition and subtraction problems. This in turn is followed by “two tape” diagrams and pairing a tape diagram with a number line. One would expect that he tape diagram or bar model, as specific examples of the diagrammatic approach, should be evident in digital mathematics textbooks developed in those countries.

Digital mathematics textbooks clearly create opportunities to harness the interactivity possible with computers to support student learning. Publishing formats such as EPUB, which supports reflowing text, or PDF, which maintains the page format, coupled with the needs of e-readers, can support or limit the interactivity needed in digital mathematics texts. To reach their potential, digital mathematics textbooks need to address the challenge of supporting the representational tools that students use to give meaning to mathematics.

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