The Context Factor in Taiwanese National Mathematics Assessment

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Prelude

Math in Context?

Question 1 (Sample Space?)

A and B have a competition of basketball shooting.

A shot 3 goals out of 5 and B shot 4 goals out of 6. Who has a higher shooting rate?

- To which grade level the question is appropriate (elementary, junior high, or senior high school)?
- Is the question reasonable?
- Is there only one answer?
- To answer this question, what additional information you need?

Question 2 (Variation?)

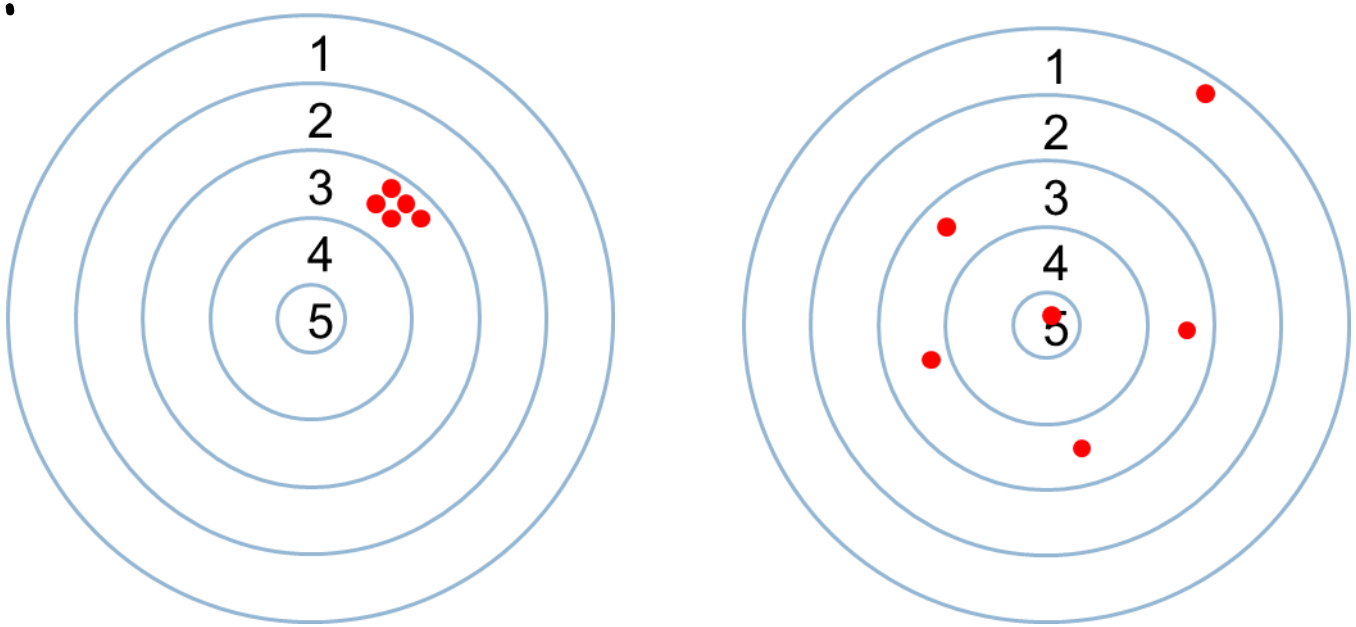
(For tracing the track, the situation is changed to throwing darts at the target of the dartboard)

A and B have a competition of dart throwing games. A threw 5 times and got 15 scores. B threw 6 times and got 18 scores. Who has a higher target throwing rate?

- To answer this question, what else information do you need? Is there only one answer?

Question 3 (Sample Space & Varation)

A and B's records on the dartboards are presented as follows:

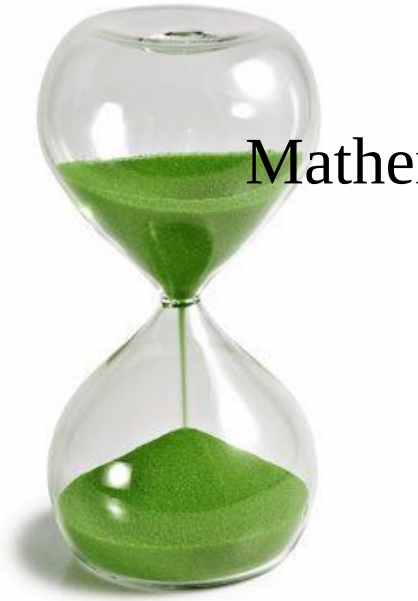


Assuming that A and B throw 10 times, please predict who get a higher score?

Outline

I. When and how we have learned the context factor
II. Context factor in college entrance examination
III. National assessment for enhancing mathematical literacy
IV. Reflection

I. When and How We Have Learned the Context Factor



Research on
Mathematical Understanding of Taiwanese Students (1985~)

Research on Students' Mathematical Understanding:
The case of understanding in **Multiple-Ratio** and **Non-Linear Ratio**

- **Background:**
  - ✓Extension of the previous research on Taiwan students' mathematical understanding of *simple ratio* (1985-1987; aged 13-15, N=3089): Levels 1-4
  - ✓Subject: 10th and 11th graders (1988; aged 16-17, **N=1539**)
  - ✓Understanding of ratio beyond level 4
- **Data Analysis**
  - ❖ 1st order analysis
  - ❖ 2nd order analysis

- **Mathematics Structure**
  - ❖ Simple Ratio: $x:y$
  - ❖ Continued Ratio: $x:y$ and $z:y$
  - ❖ Multiple Ratio: $a \propto b \times c$
    - ▪ Type Ⅰ: "$\boldsymbol{a}$" is a constant (inverse ratio)
    - ▪ Type Ⅱ: the unknown is "$\boldsymbol{a}$"
    - ▪ Type Ⅲ: the unknown is "$\boldsymbol{b}$" or "$\boldsymbol{c}$"
  - ❖ Linear & Non-linear Ratio
    - ▪ $y = ax$
    - ▪ $y = ax^2$
    - ▪ $y = ax^3$

❖ 1st order Analysis

1) Facilities of items

2) Levels of understanding in ratio

3) Frequency of students who achieved each level

4) Characteristics of levels

5) Exemplary Items in Each Level

1) Facilities of items

Facilities of items (N=1390).

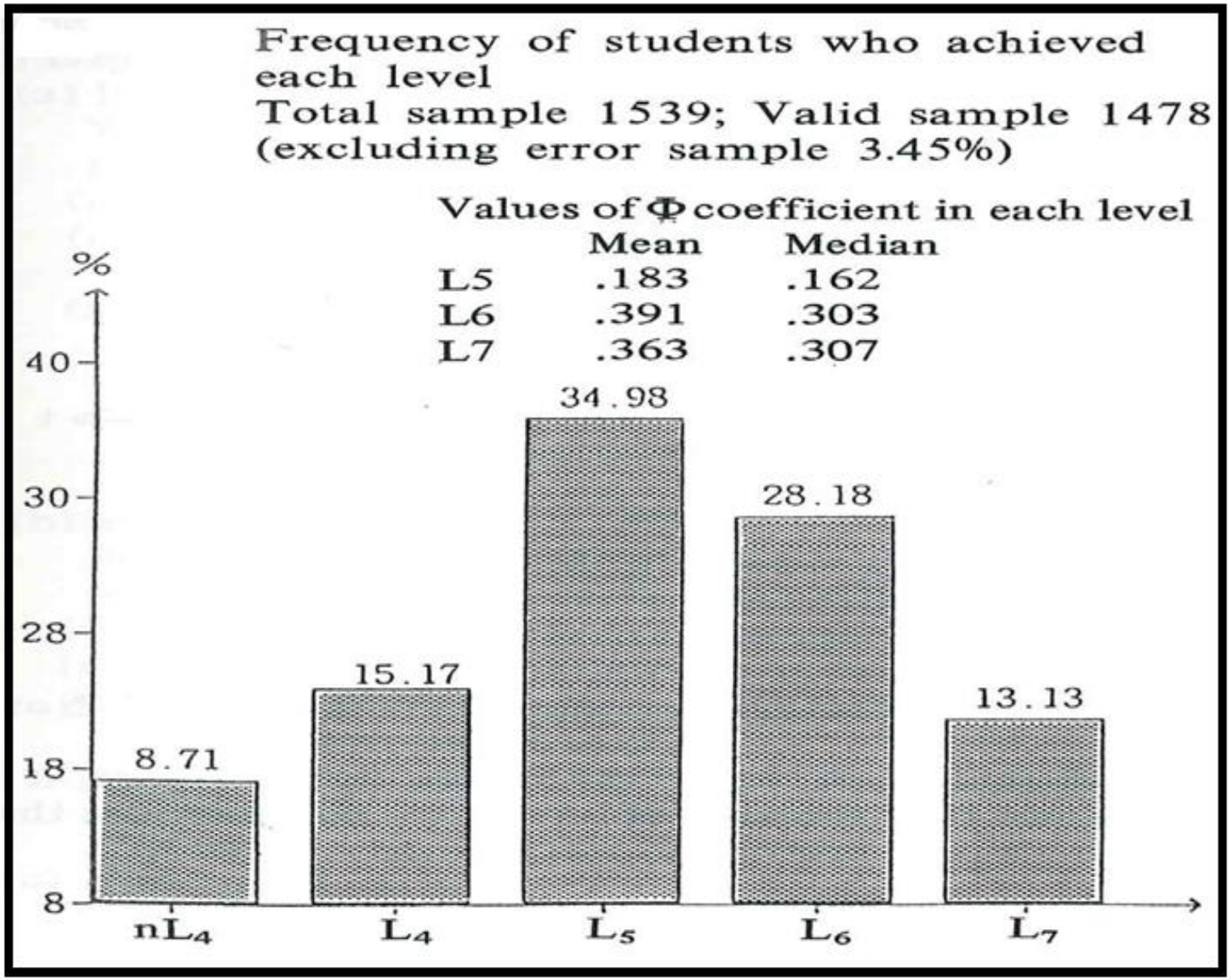
success(%)	
95.7% | 92.6%	Four simple positive ratio items with non-integer rates, e.g. 3:5, 8:12, 12:8, 6:8 etc.
82.2%	Lighting question<6(1)>, Multiple ratio: Type II (rates of change were given)
77.4%	Walking question<5>, Mulitple ratio: Type I
75.3%	Punch question<4(1)>, Continued ratio (demanding reduction of rate once)
72.9%	Bicycling question<9(1)>, Multiple ratio: Type I
67.3%	Punch question<4(2)>, continued ratio (demanding reduction of rate twice and enlargement of rate once)
65.6%	Heating question<8(1)>, Multiple ratio: Type II (rates of change must be found)
64.0%	Lighting question<6(2)>, Multiple ratio: Type III
59.8%	Heating question<8(2)>, Multiple ratio: Type III
55.6%	Recipe of Cake Question<10(1)>, $y \propto x^2$
55.4%	Motorcycling question<7>, Multiple ratio: Type I (demanding understanding of percentage)
50.3%	Recipe of cake question<10(2)>, $x \propto \sqrt{y}$
47.6%	Recipe of cake questiion<10(3)>, $x \propto \sqrt{y}$ (demanding operation on irrational number)
38.6%	New train test question<11(1)>, $y \propto x^2$
38.6%	Recipe of soup question<3(2)>, comparison of rates
34.5%	Recipe of cake question<10(4)>, comparison of rates
31.0%	New train test question<11(2)>, $x \propto \sqrt{y}$
28.6%	Bicycling question<9(3)>, Multiple ratio: Type III (Variables were described in equation form)
23.2%	Sand Timer question<12(1)>, $y \propto x^3$
18.9%	Sand Timer question<12(2)>, $x \propto \sqrt[3]{y}$

Note: Items are provided in Appendix A

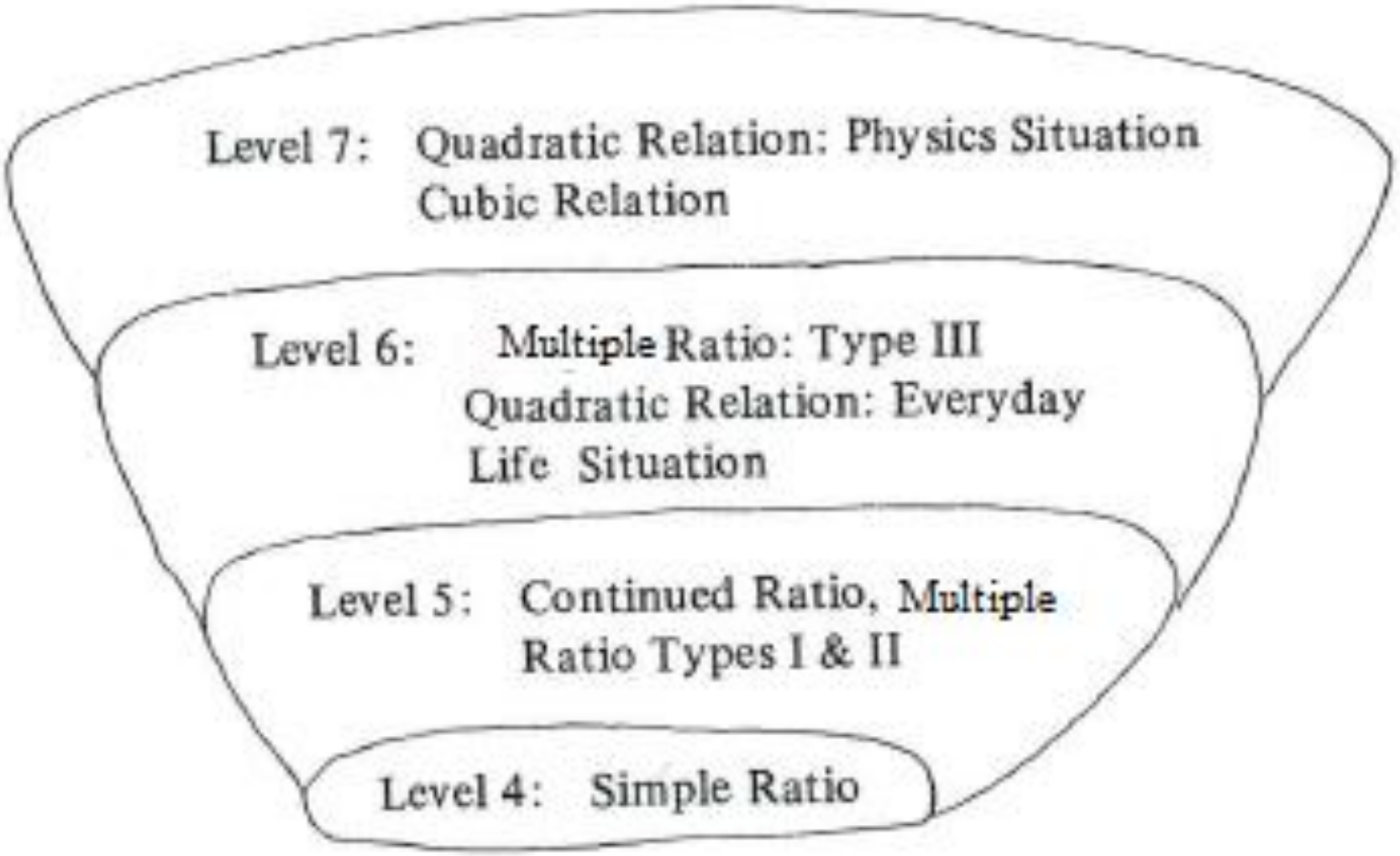
2) Levels of understanding in ratio (established statistically)

Levels of understanding in ratio		
Level	Facility Range	Pass Mark
4	92.6~95.7%	3/4
5	67.3~82.2%	3/5
6	47.5~65.6%	5/7
7	18.8~38.6%	5/7

3) Frequency of students who achieved each level



4) Characteristics of levels



5) Exemplary Items in Level 7

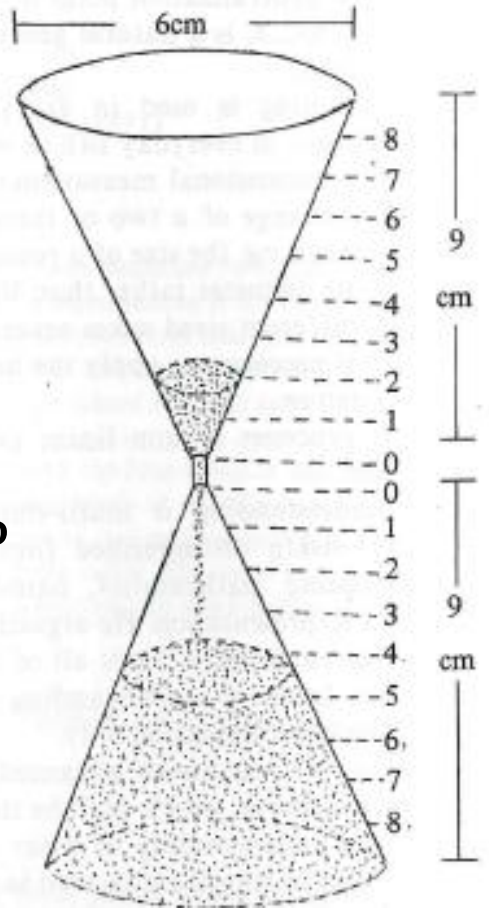
▲ **Sand Timer** (Grades 10-11; N=1390)
(12(1), non-linear Ratio: $y \propto x^3$, 23.2%, L7)
(12(2), non-linear Ratio: $x \propto \sqrt[3]{y}$, 18.9%, L7)

12.

In a sand-timer, it takes 16 secs for 2 cm of sand to drop down.

(1) How long will it take 3 cm of sand to drop down?

(2) How many cm of sand will drop down in 2 mins 8 secs?



5) Exemplary Items in Level 6

▲ **Recipe of Cake** (Grades 10-11; N=1390)
(10(1), non-linear Ratio: $y \propto x^2$, 55.6%, L6)

10.

The following recipe is for a round chocolate cake of diameter 6 inches:

7/4 cups flour 4 eggs

6 OZ chocolate 3/2 cups sugar

1/2 teaspoons soda powder

Assuming the cake has the same height,

(1) How many cups of flour do we need to make a cake with diameter 12 inches?

5) Exemplary Items in Level 5

▲ **Bicycling** (Grades 10-11; N=1390)
(9(1), Multiple Ratio:Type I, 72.9%, L5)

9.

A father and his son ride different sized bicycles to a picnic.

In 7 revolutions, the big bicycle travels the same distance as the small bicycle travels in 11 revolutions.

Also, the time the father takes to cycle 5 revolutions is the same as the son takes to cycle 8 revolutions.

(1) Find the ratio of the radius of the big bicycle to that of the small bicycle?

5) Exemplary Items in Level 5

▲ **Punch Recipe** (Grades 10-11; N=1390)
(4(1), Continued Ratio, 75.3%, L5)

4.

A punch contains four ingredients: pineapple, orange, mango and lemon.
The composition is:

With 0.6 KG pineapple, it requires 3 KG orange
With 1.8 KG lemon, it requires 6 KG orange
With 3 KG mango, it requires 9 KG orange

In this punch, considering the composition of pineapple and lemon…

With _____ kg pineapple, it requires _____kg lemon
And considering the composition of mango and lemon
With _____ kg mango, it requires _____ kg lemon

❖ 2nd order Analysis:

The Characteristics of 'Adders' in proportional reasoning

- The **West** vs. the **East**
- The **Adders** vs. the **Folding-Backers**

❖ 2nd order Analysis: The Characteristics of 'Adders' in proportional reasoning

Aspect	The England Adders (N=2257)	The Taiwan Adders (Folding-Backers; N=1599)
Multiplicative methods	• Never use them	• Use them predominantly on easy items
On easy items with ratios 1:2, 1:3	• Can cope with them by using addition-based methods, e.g. 'halving', 'doubling', 'building-up'	• Can cope with them by using algorithmic methods, e.g. 'multiplying 2 or 3', 'dividing 2', repeated addition
On hard items	• Use addition-based methods with difficulties	• Think of multiplying, however their multiplicative methods were not secure
On the addition-type questions	• View them as natural addition-type questions (to make enlargement means to add)	• Switch to incorrect-addition strategy because of, e.g. non-awareness of non-integer multiple, obstacles of the geometric settings
Fraction	• Avoid using multiplication of fractions and the taught algorithm • Poor understanding	• Can do operations of fractions by taught algorithms (learning by rote) • Manipulating them is not a ready skill • Poor understanding
Process of solving problems	• Use inconsistent methods—ask themselves "what would be most useful here?"	• Use multiplicative methods consistently by some of them • Inappropriate, divorced from understanding • "Capture effect"
Non-integer multiple	• No evidence	• Non-awareness
Within and between ratios	• No evidence	• Aware of one kind of ratio only
Distinguishing contexts	• No evidence	• poor

II. Context Factor in College Entrance Examination

1. Non-Awareness of Context – Study on ISA (1987-1991)
2. Development of Situational Items (1994~)
3. Implementation (2005-2009)
4. Discontinuity: Folding-back in No Situation (2010-2014)
5. The Context Shock (2015)

Data from the College Entrance Examination Center (CEEC)

1. Study on Item Structure Analysis (ISA) in 1987-1991– No Context Consideration in the Study

- ISA for monitoring the stability of items in each paper
- 3 variates of items:
  - *mathematical topic*
  - *thinking strategy*: mechanical, multiple, flexible
  - *extent of solution process (e.g. number of solution steps)*
- The context is not aware in the items

2. Pioneer Project in Developing Situational Items (1994~)

Workshops for Designing Situational Items

- Two kinds of context:
  - Transformed context
  - Authentic context
- Index for well situational items
  - Provide familiar context for students
  - Avoid decorated question, i.e. mathematics problem in the decoration of context
  - Prevent question in suitable context but not in real

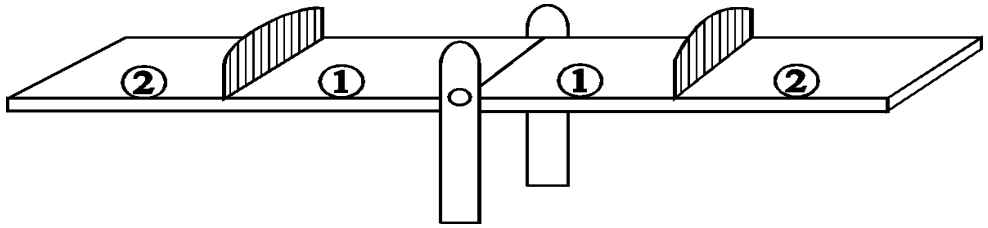
2. Pioneer Project in Developing Situational Items (1994~)

Study of Students' Performances on Situational Items

- Pilot study for National Entrance Examination with senior high school students
- Two examples in 1998 (Seesaw) and 1999 (Manhatton Distance)

▲ Seesaw (1998) – Situational Item vs. Algebraic Item (N=136)

A competition of seesaw between two teams.



(1) Each team include two people of different weights. Which combination is the best strategy to win for a specific team?

Score	0	1	5
%	6	54	40

(2) Each team include three people of different weights. Which combination is the best strategy to win for a specific team?

Score	0	2	10
%	10	60	30

(3) Given $0 < m_1 < m_2 < m_3$, $0 < d_1 < d_2 < d_3$, and $\{i, j, k\} = \{1, 2, 3\}$ ◦ To find the biggest $m_1 d_i + m_2 d_j + m_3 d_k$ among all combination of i, j, k and explain it.

Score	0	1	5
%	40	36	24

▲ Seesaw

Interviews with Students and Teachers

Students

Question: Is there any disturbance in understanding the mathematical problem?

- No: 54%; Yes: 38%
- The elaboration of the mathematical problem is not clear. For example, if the heaviest person in Team A is even lighter than any one in Team B, there is no possibility to win for Team A.
- I don’t know whether the people still can move after they set their positions.
- Is the interval between seats the same distance?

Teachers

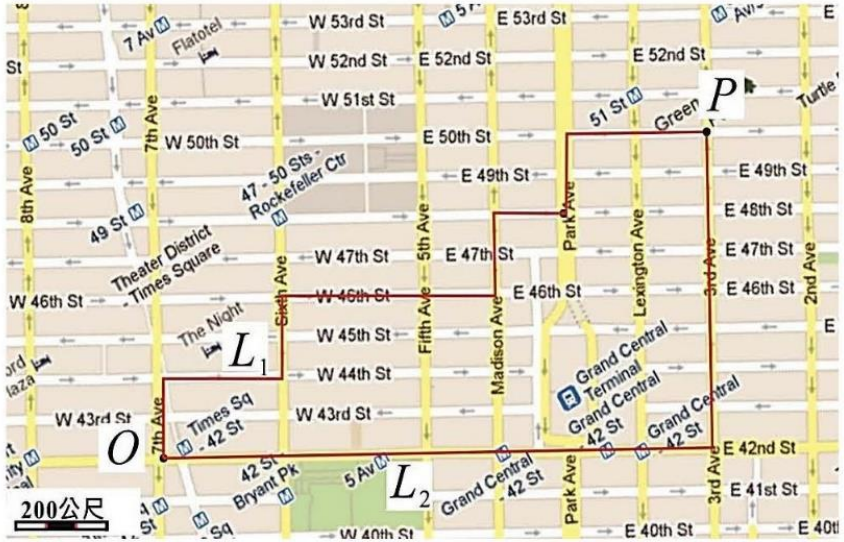
- This version of items is much suitable for the Advanced Subject Test.
- Such item can help student to make reasonable judgement on daily life events embedded mathematical properties through mathematical thinking.

▲ Manhattan Distance (1999)

Version 1 (Authentic Context, N=503)

- To describe/understand
- To explain
- To predict

一、圖一為曼哈頓某一區的地圖。由於紐約曼哈頓地區街道大部分是由水平和鉛直路線形成，當地計程車司機談論兩地點的距離，不是指此兩地間的直線距離，而是指兩地水平位移距離和鉛直位移距離之和，此稱為曼哈頓距離。例如 A 點的坐標為 $(100,700)$，B 點的坐標為 $(300, 20)$，則 A、B 兩點的曼哈頓距離為 $|300-100|+|-200-700|=200+900=1100$。



圖一

問題(1)：

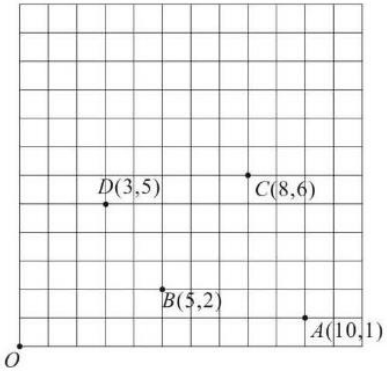
估計圖一上 O、P 兩地點的曼哈頓距離。並且判斷計程車分別行駛 L_1、L_2 兩條路徑所走的距離，是否為 O、P 兩地點的曼哈頓距離。

問題(2)：在圖二，已知 Q、X 兩點的曼哈頓距離為 1400 公尺。請問 $Q_1,\ldots,Q_{10}$ 中，有哪些點和 X 的曼哈頓距離也是 1400 公尺？請說明理由。

圖二

問題(3)：承上題，在那些與 X 的曼哈頓距離是 1400 公尺的點中（含 Q 點），請問哪一點和 X 的<u>直線距離</u>最短？

問題(4)：在圖三中，計程車司機從 O 出發，依序到 A、B、C、D 再回到 O。如果司機在兩點間行駛的距離皆為曼哈頓距離，請問行駛此路徑的距離為何？



圖三

問題(5)：承上題，司機從 O 出發，要經過 A、B、C、D 這四點（但順序不拘），再回到 O。請選出一個比上題中所選路徑的距離更短的路徑順序。

▲ Manhattan Distance (1999)

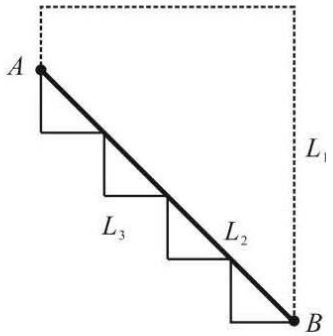
Version 2 (Transformed Context, N=516)

- To describe/understand
- To explain
- To predict

二、在坐標平面上，兩點的曼哈頓距離定義為：此兩點所成的向量其水平分量的長度和垂直分量的長度之和。例如 O 點的坐標為 $(0,0)$，P 點的坐標為 $(30,-20)$，那麼 O、P 兩點的曼哈頓距離為

$|30-0|+|-20-0|=30+20=50$。

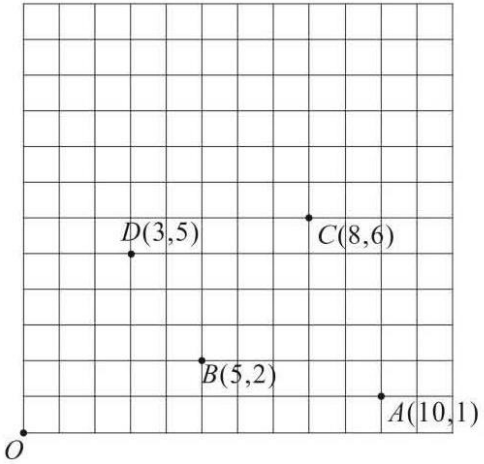
問題(1)：已知 $A(20,8)$、$B(-32,$ 兩點在坐標平面相關位置如右。試求 A、B 兩點的曼哈頓距離，並說明 L_1、L_2、L_3 這三條路徑中，哪一條路徑所經距離恰為 A、B 兩點的曼哈頓距離。



問題(2)：承上題，請求出 x 軸上哪些點和 A 的曼哈頓距離會和 A、B 兩點的曼哈頓距離相同，並說明理由。

問題(3)：在 x 都20, y 8的範圍中，與 A 的曼哈頓距離會和 A、B 兩點的曼哈頓距離相同並與 A 的直線距離最短的點坐標為何？

問題(4)：下圖中，從 O 出發，依序到 A、B、C、D 再回到 O。如果兩點間的距離皆以曼哈頓距離計算，請問此路徑的距離為何？



問題(5)：承上題，從 O 出發，要經過 A、B、C、D 這四點（但順序不拘），再回到 O。請選出一個比上題中所選路徑的距離更短的路徑順序。

▲ Manhattan Distance
Questionnaires collected from Students

Question 1:

The reason why you think the difficulty is higher than the normal items?

	(*f*/%)
(1) The statement is too long to read	54/36%
(2) The statement is not clear enough	77/51%
(3) There are too many redundant words	30/20%
(4) Others	71/47%

▲ Manhattan Distance
Questionnaires collected from Students

Question 2:

Why you think the difficulty is lower than the items in the past years? (multiple choices)

	(*f*/%)
(1) The description of situation is clear and easy to touch	46/38%
(2) The expression of statement is easy to understand	70/57%
(3) The guidance of the question is described clearly	42/64%
(4) Others	32/34%

▲ Manhattan Distance
Questionnaires collected from Students

Question 3:

Do you think 20 minutes is enough to answer this item?

	(*f*/%)
(1) Absolutely yes. There's still enough time to check.	58/12%
(2) Yes. Time is right enough to answer.	173/35%
(3) No. Time is only enough for half item.	201/40%
(4) No. Time is only enough for reading the item. No time left for answering.	30/6%

3. Implementation

- Situational items are popular in the College Entrance Examination in the first decade of the 21st century

4. Discontinuity: Folding-Back (2010-2014)

- The national entrance examination
  - The number of situational items is decreasing

Year	Number of Situational Item	
	Science track	Social science track
2010	2	2
2011	1	3
2012	2	2
2013	2	3
2014	3	0

- Item developers changed!

5. Context Shock from the College Entrance Examination (2015)

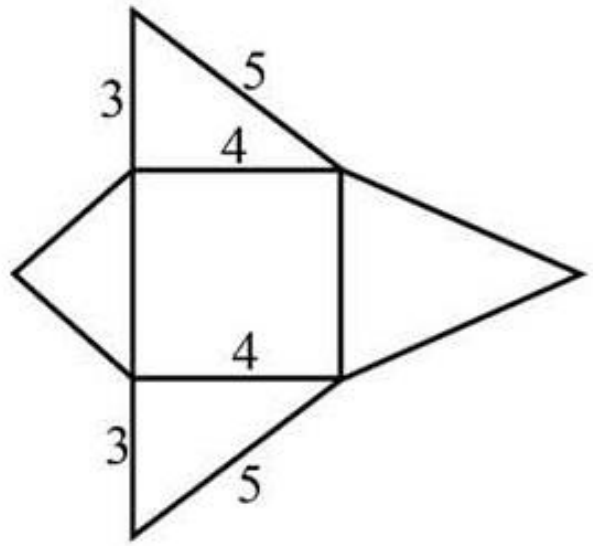
- 70% (14/20) of situational item in 2015 College Entrance Examination paper

- Items with Fresh Situations:
  - **Wordy** questions
  - Degree of **Familiarity**

Fresh Situation

▲ The Volume of an Unfolded Box (N=144,061)

有一底面為正方形的四角錐，其展開圖如下圖所示，其中兩側面的三角形邊長為 3,4,5，則此角錐的體積為 $\frac{㉙㉚\sqrt{㉛}}{3}$。（化為最簡根式）



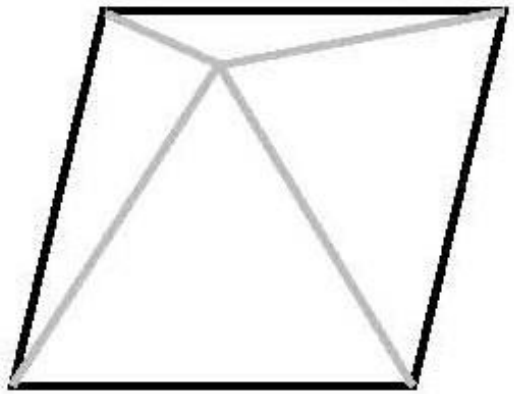
P=43%, D=.66

Fresh Situation

▲ The Pyramid (N=144,061)

To find the Skew Angle of any two faces of a given regular-based Pyramid with slope of $\frac{2}{5}$

在空間中，一個斜面的「坡度」定義為斜面與水平面夾角 θ 的正切值 $\tan\theta$。若一金字塔（底部為一正方形，四個斜面為等腰三角形）的每一個斜面的坡度皆為 $\frac{2}{5}$，如圖。則相鄰斜面的夾角的餘弦函數的絕對值為 $\frac{\textcircled{32}\,\textcircled{33}}{\textcircled{34}\,\textcircled{35}}$。（化為最簡分數）



P=13%, D=.33

Wordy Question

▲ The Quarter Marathon (ten kilometers) (N=144,061)

completing time, average distance of each step, average of heart pulse, number of steps, the correlations

小明參加某次路跑 10 公里組的比賽，下表爲小明手錶所記錄之各公里的完成時間、平均心率及步數：

	完成時間	平均心率	步數
第一公里	5:00	161	990
第二公里	4:50	162	1000
第三公里	4:50	165	1005
第四公里	4:55	162	995
第五公里	4:40	171	1015
第六公里	4:41	170	1005
第七公里	4:35	173	1050
第八公里	4:35	181	1050
第九公里	4:40	171	1050
第十公里	4:34	188	1100

在這 10 公里的比賽過程，請依據上述數據，選出正確的選項。

(1) 由每公里的平均心率得知小明最高心率爲 188

(2) 小明此次路跑，每步距離的平均小於 1 公尺

(3) 每公里完成時間和每公里平均心率的相關係數爲正相關

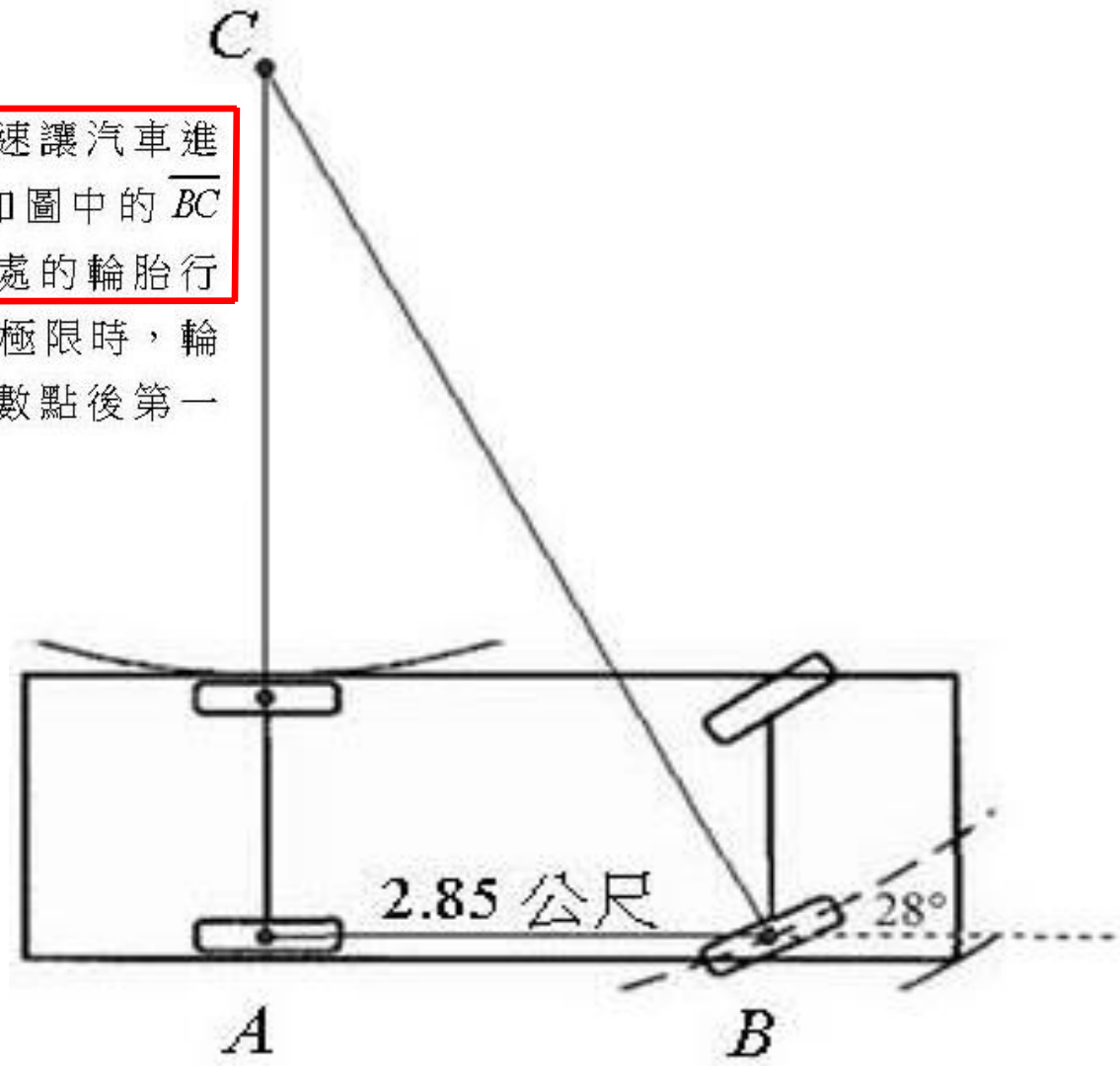
(4) 每公里步數和每公里平均心率的相關係數爲正相關

(5) 每公里完成時間和每公里步數的相關係數爲負相關

P=48%, D=.56

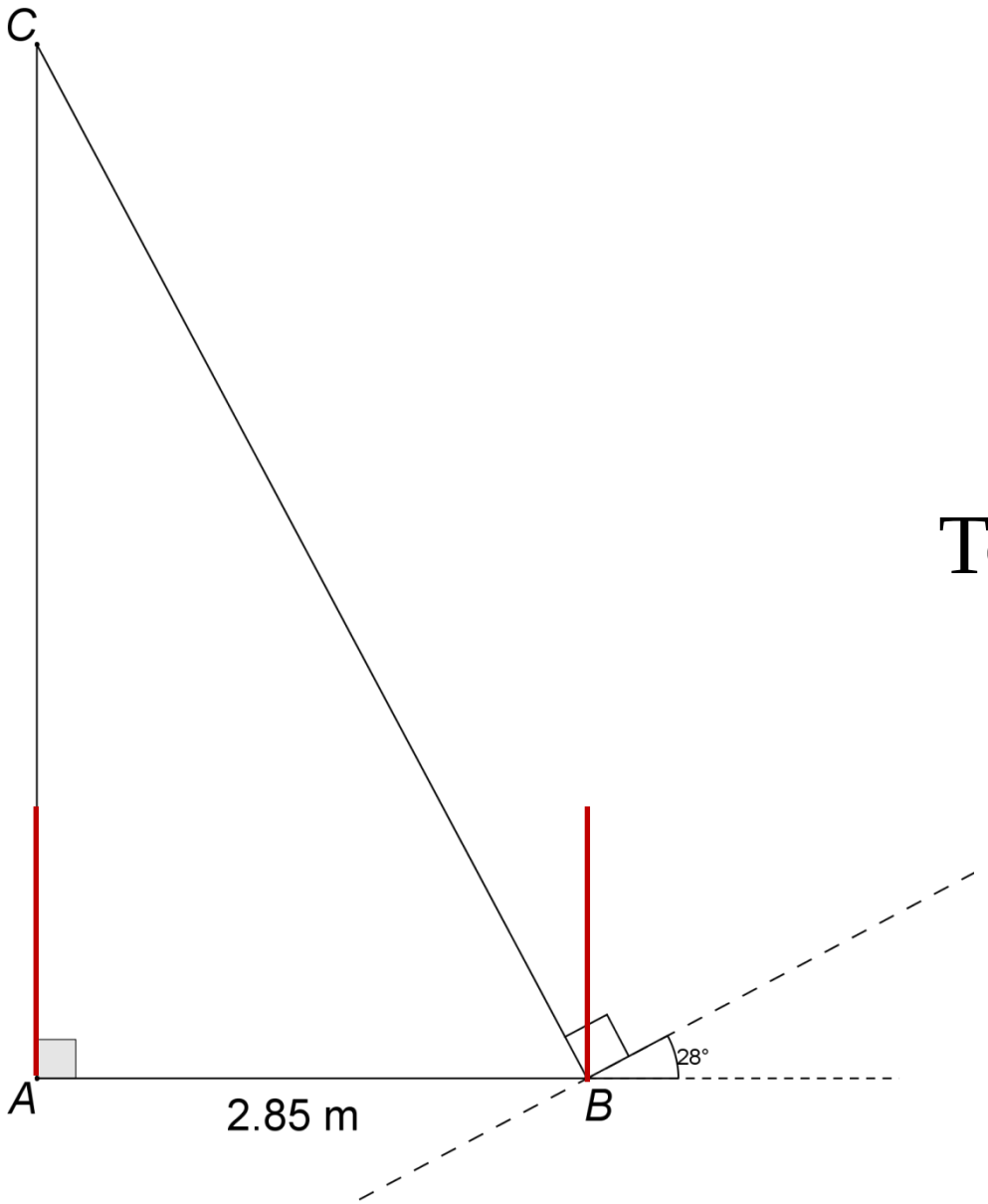
▲ The Turning Radius of a Car (unfamiliar) (N=144,061)

下圖為汽車迴轉示意圖。汽車迴轉時，將方向盤轉動到極限，以低速讓汽車進行轉向圓周運動，汽車轉向時所形成的圓周的半徑就是迴轉半徑，如圖中的 $\overline{BC}$ 即是。已知在低速前進時，圖中 A 處的輪胎行進方向與 $\overline{AC}$ 垂直，B 處的輪胎行進方向與 $\overline{BC}$ 垂直。在圖中，已知軸距 $\overline{AB}$ 為 2.85 公尺，方向盤轉到極限時，輪子方向偏了 28 度，試問此車的迴轉半徑 $\overline{BC}$ 為 ⑯.⑰ 公尺。（小數點後第一位以下四捨五入，$\sin 28° \approx 0.4695$, $\cos 28° \approx 0.8829$）



P=32%, D=.69

▲ The Turning Radius of a Car (unfamiliar) (N=144,061)



To find $\overline{BC}$ =?

III. National Assessment for Enhancing Mathematical Literacy

1. Initiation of PISA-Like Items Designing: Two volumes resulted from workshops in 2010-2011
2. PISA Storming (2012~)

1. PISA-Like Items Designing:
Two volumes resulted from the designing workshops in 2010-2011

Published Books of PISA-Like Items in 2011 (Lin, Ed.)

- 《臺灣2011 *數學素養*評量樣本試題》Vol.1: http://www.most.gov.tw/sci/public/Attachment/282115164871.pdf
- 《臺灣2011 *數學素養*評量樣本試題》Vol.2: http://www.most.gov.tw/sci/public/Attachment/282115172171.pdf
- Contributors:
  - 25 Professors in the domains of mathematics and mathematics education
  - 180 Junior High School Mathematics Teachers
- Item Categories and Numbers:
  - Personal Situation: 58 items
  - Educational/Vocational Situation: 31 items
  - Societal Situation: 64 items
  - Scientific Situation: 27 items

第二章 個人情境 試題

零、總表

編碼	題目名稱	個人情境				頁數
		變與關係	空間與圖形	量	不確定性	
P001	大智慧過生活		√	√		61
P002	分食披薩		√		√	149
P003	太陽能熱水器	√		√		9
P004	手機			√		99
P005	手機費率			√		101
P006	手續費	√		√		12
P007	出遊規劃			√	√	151
P008	加油也要精打細算！	√		√		14
P009	加法運算	√		√		17
P010	台灣高鐵			√		104
P011	永保安康		√	√		64
P012	用呼拉圈量長度			√		107
P013	田徑場		√			66
P014	交通號誌	√		√		19
P015	回收寶特瓶	√				21
P016	安全開門		√	√	√	154
P017	成績的計算			√	√	156
P018	羊肉爐材料數量	√		√		24
P019	自助旅行			√		109
P020	我是數字王	√		√		26
P021	披薩大小		√	√		69
P022	促銷折扣	√		√		28
P023	省電燈具	√		√		30
P024	省錢達人			√		111
P025	美麗人生	√		√		33
P026	胡椒餅	√				38
P027	要如何救貓？	√	√	√		71
P028	重量誤差			√	√	158
P029	食物熱量			√		114
P030	旅行		√	√	√	160
P031	旅行社			√		116
P032	均分桂圓蛋糕			√		118
P033	高雄捷運			√		121
P034	排隊問題	√		√		40

2-1

編碼	題目名稱	個人情境				頁數
		變與關係	空間與圖形	量	不確定性	
P035	涼亭		√			74
P036	畢旅好好玩	√		√		42
P037	眼鏡		√			77
P038	蛋糕食譜		√	√		79
P039	速食店的套餐	√		√		45
P040	超值餐	√		√		47
P041	飲料折扣			√		123
P042	搭公車			√		125
P043	碗的疊放		√			81
P044	跨年夜		√	√	√	162
P045	夢想中的夢想			√		127
P046	摺紙		√	√		83
P047	漁港		√	√		86
P048	影印機		√			88
P049	摩天輪			√		136
P050	膠帶	√		√		52
P051	選擇			√		139
P052	餐廳優惠	√		√		54
P053	優惠問題			√		141
P054	營養配餐	√		√		56
P055	骰型		√	√		96
P056	購物	√		√		58
P057	點水成金			√		143
P058	雙雙對對的長條圖			√		146

註 1：總表編碼規則 【個人（person）情境編為 P】；【教育/職業（education/occupation）編為 E】；【社會（公民）（citizen）編為 C】；【科學（science）編為 S】，故總表的編碼為「情境-題號」，例如：P006（個人情境-第六題）。

註 2：題目編碼規則 知識大客分為四類，【變與關係（change and relation）編為 R】、【空間與形（space and graph）編為 S】、【量（quantity）編為 Q】、【不確定性（uncertainty）編為 U】，故題目編碼方式為「情境-內容-題號」，例如：P-RQ-023（個人情境-變與關係、量-第 23 題）。

2. PISA Storming (2012~)

1) See

Yang, K.-L., & Lin, F.-L. (2015). The effects of PISA in Taiwan: Contemporary Assessment Reform. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy: The PISA Experience* (pp. 261-273). Springer.

2) Following Workshop for Situational Item Designing

- **Strategy**-used in item writing workshop
- **Big idea** for item writing

3) PISA-Like Items Bank:

http://140.122.140.2/museum/Test/Joomla_2.5.24_2/

1) The effects of PISA in Taiwan

In Taiwan, PISA used to be an inactive seed
Appears once every three years
Could only be seen in newspapers.
Taiwanese performance in PISA
Seemed to be similar in TIMSS.
Be excellent in mathematics and science literacy
But poor in reading literacy.
After summer 2012, the inactive seed suddenly burst
Into every family with high school students,
Into the minds of all high school teachers,
Into daily conversations of Taiwanese educational community.
Meanwhile, a strange phenomenon arose:

PISA cram schools shot out numerously.

1) The effects of PISA in Taiwan

Background

- Educational system reform (12-year compulsory education)
- Needs of an additional Entrance Examination for recruiting talented senior high school students
- PISA-like assessment is recommended

A Report from the Newspaper

(Article from the China Times of 1 January 2013) (Yang & Lin, 2015)

PISA Assessment for Entrance Examination in Keelung and Greater Taipei: Parents Are Much More Worried than Students (1)

- The Chairman of the Secondary School Parents Association in Taipei, Mr. Young-Jia Hsu, has criticised some buxiban that take advantage of the panic and anxiety of parents and students to recruit students into PISA training sessions. No matter what the effects are, this situation is similar to fraud.

1) The effects of PISA in Taiwan: Needs of an additional Entrance Examination

PISA Assessment for Entrance Examination in Keelung and Greater Taipei: Parents Are Much More Worried than Students (2)

- The reporter visited several buxiban with PISA training sessions in Taipei and found the cost is about NTD$650-850 per lesson, which is up to 10% higher than general courses at the buxiban.

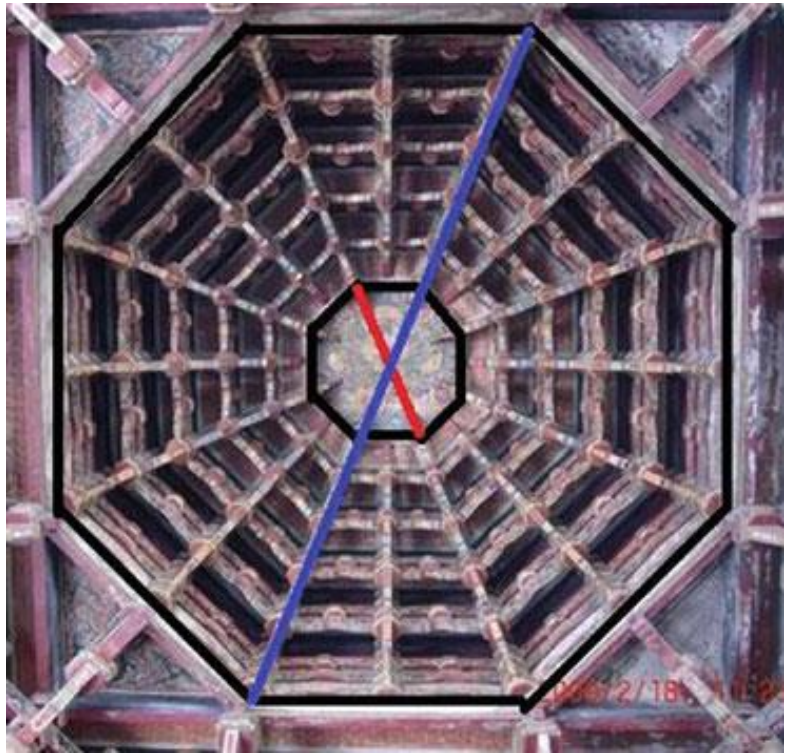
1) The effects of PISA in Taiwan: Needs of an additional Entrance Examination

PISA Assessment for Entrance Examination in Keelung and Greater Taipei: Parents Are Much More Worried than Students (3)

- The Deputy Chief of Department of Education, Taipei City Government, Dr. Ching-Huang Feng states that the Comprehensive Assessment Program for Junior High School Students includes the traditional five courses in the assessment. If the special high school admission examination takes the same courses as well, Taiwanese secondary teaching will follow our old route only emphasizing memorisation and repeated practice. However, the Keelung and Greater Taipei regions will include literacy courses in the assessment as has been publicized widely. Without this, it would be difficult to make any change. Therefore, the Department of Education, Taipei City Government has requested schools to include literacy questions in general assessments for Grade 7 and 8 students, in the hope that students will gradually become familiar with questions of this kind, and so be confident when participating in the special high school admission examination.

▲ The Eight Trigrams Shaped Ceiling Problem for 9th graders

(Yang & Lin, 2015)



Here is the Eight Trigrams shaped ceiling of Lu-Gang Longshan Temple, the biggest ceiling in Taiwan. Its span, the diagonal line shown over the outermost layer of the octagon, is about 5.5 feet and the height of the top centre is about 6.5 feet. It is tiered up with five layers, each made of 16 crossbeams to support the weight of the roof eaves. The crossbeams are carved with exquisite sculptures from Chinese culture. The ceiling is built using nails of wood rather than metal. The ceiling is filled with the wide and deep wisdom of our ancestors.

Please estimate the length of the diagonal line in the innermost layer of the octagon.

(1 foot = 12 inches, 1 inch = 2.54 cm)

1) The effects of PISA in Taiwan: PISA-like Assessment

▲ The Buxiban Advertisement Problem (Yang & Lin, 2015)

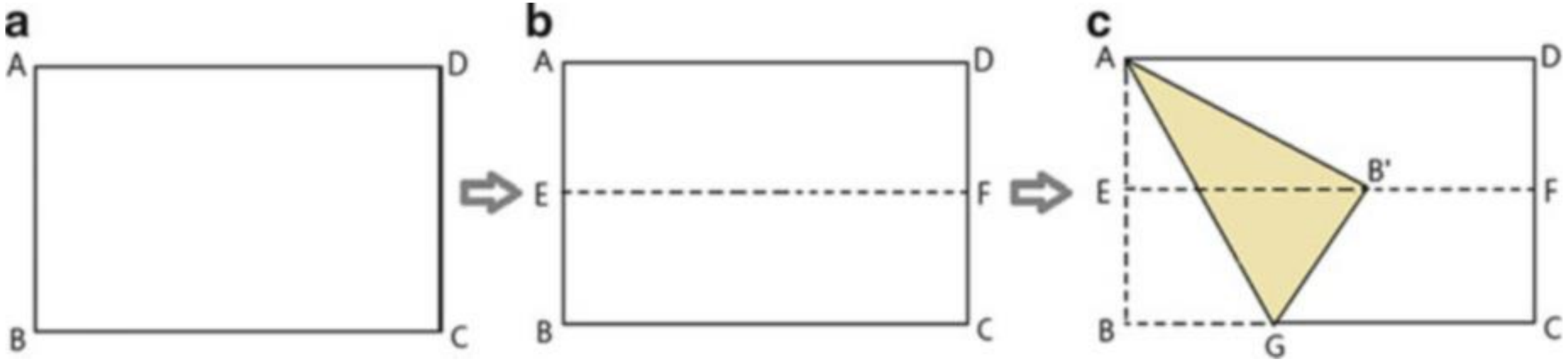
There are three competitive buxiban in Ting-Sou's hometown. The number of students attending the buxiban and the number of these passing the Basic Competence Test for Junior High School Students are shown in the table for the past three years Because of the keen competition among these three buxiban, if they ever use any false data, that buxiban will be attacked by the other two. Consequently, the buxiban will lose its credit and students, and have to pay a fine for false advertisement. Therefore, all the buxiban use real data to design favourable flyers for themselves. Please answer the following questions using the data in the table.

If you are the publicity manager of one of these buxiban, how would you design a statistical chart to highlight the advantage of your company? (Provide an answer for each buxiban.)

Buxiban	96th academic year		97th academic year		98th academic year	
	Number of students	Number of passes	Number of students	Number of passes	Number of students	Number of passes
P	60	30	65	31	80	32
S	130	40	120	40	125	40
Q	35	12	42	15	39	15

1) The effects of PISA in Taiwan: PISA-like Assessment

▲ The Paper-folding Problem (Yang & Lin, 2015)



2) Following Workshop for Situational Item Designing

- **Strategy-used in item writing workshop**
  - Design following Given Context

- **Big Ideas for item writing**

- Multiple Interpretation - Critical Comments - Definition	• To Describe/Understand • To Explain • To Predict

2) Following Workshop for Situational Item Designing

Note: Corresponding Given Situation see Appendix B

Designs Following Given Situation Structure

Societal and Developmental Trend

- Sobriety Test
- Territorial Sea
- Higher Degree, Higher Income
- The Median of Age among Citizens
- Earthquake
- Fixation on the Billboard

Personal

- BMI
- Quotient of BMI
- Board Game

Vocational

- Austinogebia edulis (mud shrimp)
- The Blueprint
- Semicircle Glass Cutting
- Paper Folding

Scientific

- The Rolling Coin
- Water-saving Rationale for Toilet
- The Squareness of the Quadrilateral

3) PISA-Like Item Bank http://140.122.140.2/museum/Test/Joomla_2.5.24_2/

3) PISA-Like Item Bank http://140.122.140.2/museum/Test/Joomla_2.5.24_2/

IV. Reflection

Educational Reforming

Educational Reforming

1. Matching in-school assessment with entrance examination
2. TPD: Designing situational items as a strategy for in-service mathematics teachers professional development
3. Actions: Lighten-Up School-Based Project (LUSBP)

In Acknowledgement of

Dr. Yu-Ping Chang preparing the slides for this presentation

Appendix A: Items of Multiple-Ratio and Non-Linear Ration (1)

<5> Ming and Huei go to school by the same route. Ming leave at 7:05. Huei leave at 7:12. Huei catches up with Ming at 7:25. Find the ratio of Ming's average speed to Huei's average speed?

<6> The power consumption of some bulbs during a period is 6 units.

(1) If the number of bulbs is halved, and the time period increased 3 times, find the number of power units consumed?

(2) If the number of bulbs is increased 4 times, and 4 power units are consumed, find how long the bulbs are alight?

<7> The distance between town A and B is 72 km. It takes Ming 1 hr 54 min. to go from A to B by motorcycle. On the return journey, Ming increases his speed by 20%. How long does the return journey take?

<8> To increase the temperature of 50L of water by 5.7°C requires 30L of gas.

(1) How much gas does it take to increase the temperature of 25L of water by 7.1°C?

(2) If 20L of gas are used to heat 200L of water, by how much will its temperature increase?

Appendix A: Items of Multiple-Ratio and Non-Linear Ration (2)

<9> A father and his son ride different sized bicycles to a picnic. In 7 revolutions, the big bicycle travels the same distance as the small bicycle travels in 11 revolutions. Also, the time the father takes to cycle 5 revolutions is the same as the son takes to cycle 8 revolutions.

(1) Find the ratio of the radius of the big bicycle to that of the small bicycle?

(3) Find the ratio of the speed of the big bicycle to that of the small bicycle?

<11> A new train is testing its speed limits. To do this, a time-maker is placed every 100 km. The train travels with constant acceleration, and the first time marker shows 1.28 secs.

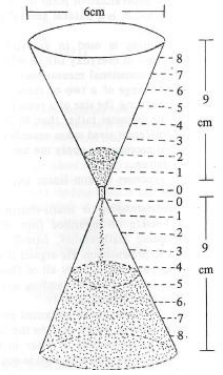
(1) How far will the train have travelled after 2.56 secs from the start?

(2) What time will be shown by the 8th time-maker?

<12> In a sand-timer, it takes 16 secs for 2 cm of sand to drop down.

(1) How long will it take 3 cm of sand to drop down?

(2) How many cm of sand will drop down in 2 mins 8 secs?



Appendix B
Given Situation Structure

Sobriety Test at a Roadside Checkpoint

警方在路邊對駕駛人進行酒駕臨檢時，通常以吹氣方式檢測呼氣濃度是否超標。

一個人到底喝多少酒開車上路才不會被罰(呼氣酒精濃度達到0.25mg/L)呢？

以下計算方式可約略簡化為與體重及酒類兩個主要因素相關，作為個人簡易判斷的依據：安全飲酒量(cc) = 體重(kg) * 0.8 / 酒精濃度(%)

另外，法醫研究所有套酒測值的回推公式，根據人體代謝100cc血液，每小時會下降10到20毫克的酒精濃度，經過換算，大約等於呼氣的酒精濃度，每小時會下降0.1毫克，不管酒駕駕駛怎麼拖延時間，只要一推算，都能還原之前的酒測值，愛喝酒的人千萬別心存僥倖。

BMI

營養上的理想體重以促進健康和增長壽命為標準。由於個人的體型不同，常用的參考指標是身體質量指數（Body Mass Index，縮寫為BMI），其計算方式為：

BMI = 體重 (kg) / 身高平方 ()，體重的單位為公斤，身高的單位為公尺。

成人的體重分級與標準	
分　級	身體質量指數
體重過輕	BMI ＜ 18.5
正常範圍	**18.5 ≦ BMI ＜24**
過　重	24 ≦ BMI ＜ 27
輕度肥胖	27 ≦ BMI ＜ 30
中度肥胖	30 ≦ BMI ＜ 35
重度肥胖	BMI ≧ 35
資料來源：衛生署食品資訊網／肥胖及體重控制	

Quotient of BMI

身體質量指數（Body Mass Index，縮寫為BMI）計算方式如下：

BMI = 體重 (kg) / 身高平方 ()，體重的單位為公斤，身高的單位為公尺。

成人的體重分級與標準	
分　級	身體質量指數
體重過輕	BMI ＜ 18.5
正常範圍	**18.5 ≦ BMI ＜24**
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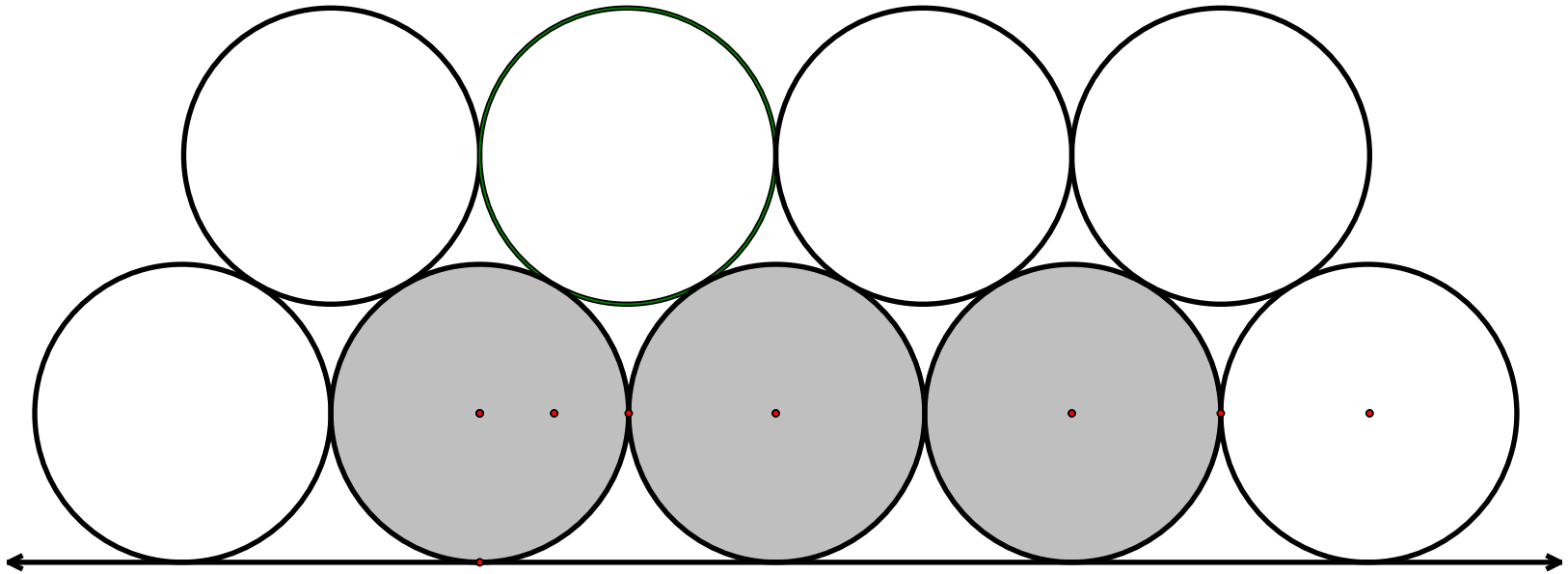
中度肥胖	30 ≦ BMI ＜ 35
重度肥胖	BMI ≧ 35
資料來源：衛生署食品資訊網／肥胖及體重控制	

Board Game (The game of Go)

小明在圍棋的黑子與白子的搶地中，知道要將愈多的黑子圍起來，就需要愈多的白子。設表黑子的數目，表白子的數目。



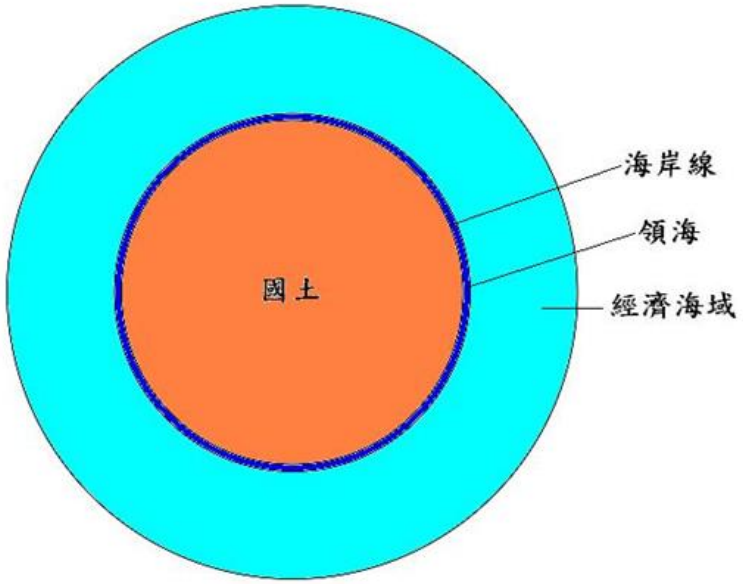
Territorial Sea -1

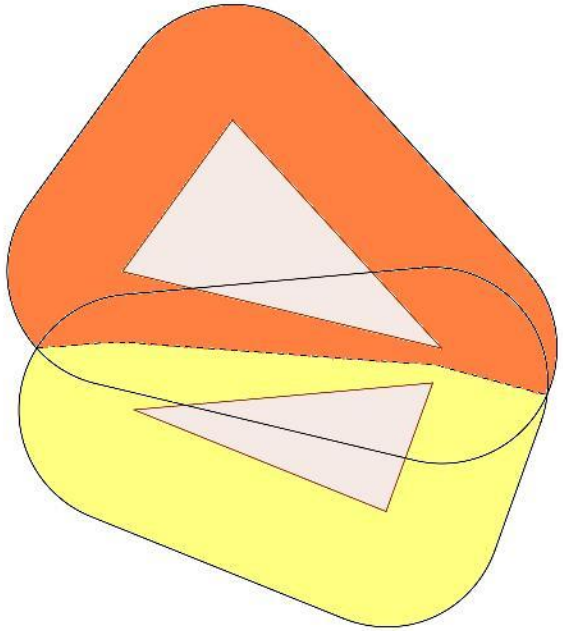
2013年5月9號，我國漁船「廣大興28號」，遭菲律賓海巡署公務船以機槍掃射，造成屏東縣漁民洪石成中彈身亡。此事件造成台、菲兩國情勢緊張，而且對於海權之爭的議題也引起熱烈討論。

根據聯合國海洋法公約（United Nations Convention on the Law of the Sea，簡記為UNCLOS），各國可主張海岸線沿岸12浬範圍內屬於領海、海岸線沿岸200浬範圍內屬於經濟海域。各國可在其經濟海域內進行符合國際法規的資源開發與使用（例如鋪設海底電纜，以及開發礦產、漁業資源等）。

Territorial Sea -2

例如下左圖，某國國土是一個圓形，中間環狀區域為領海，最外圍環狀區域為經濟海域。

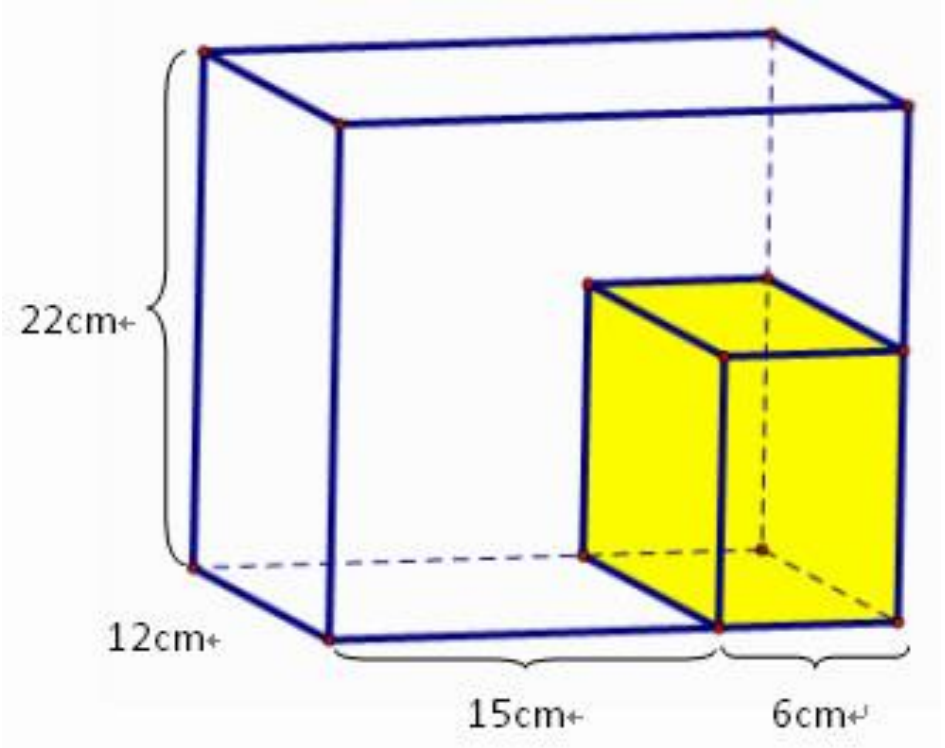




如果兩國領海或經濟海域有重疊的區域，原則上以中線劃分，或由兩國協議訂定。所謂中線，意思是到兩國國境最短距離相等的點所構成的邊線。例如上右圖。

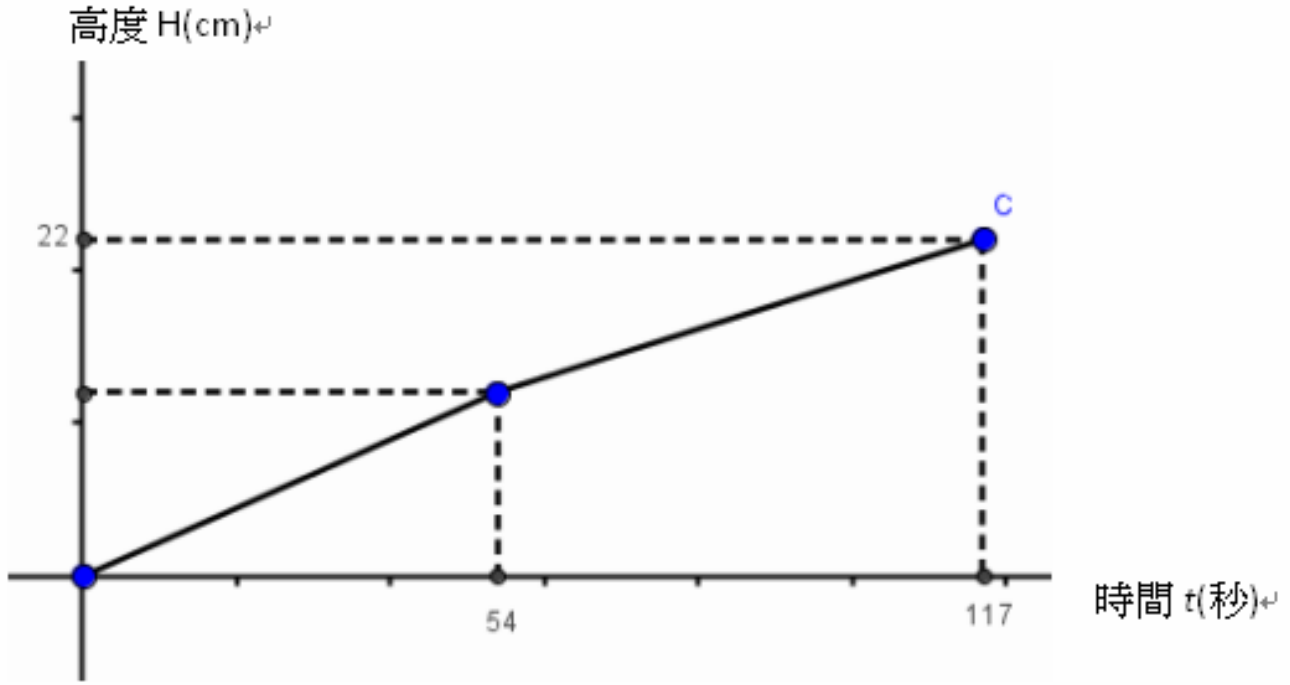
Water-saving Rationale for Toilet -1

為了達到環保省水的目標，在長方體馬桶水箱的角落放入一塊長方體磚頭，如下圖所示：



Water-saving Rationale for Toilet -2

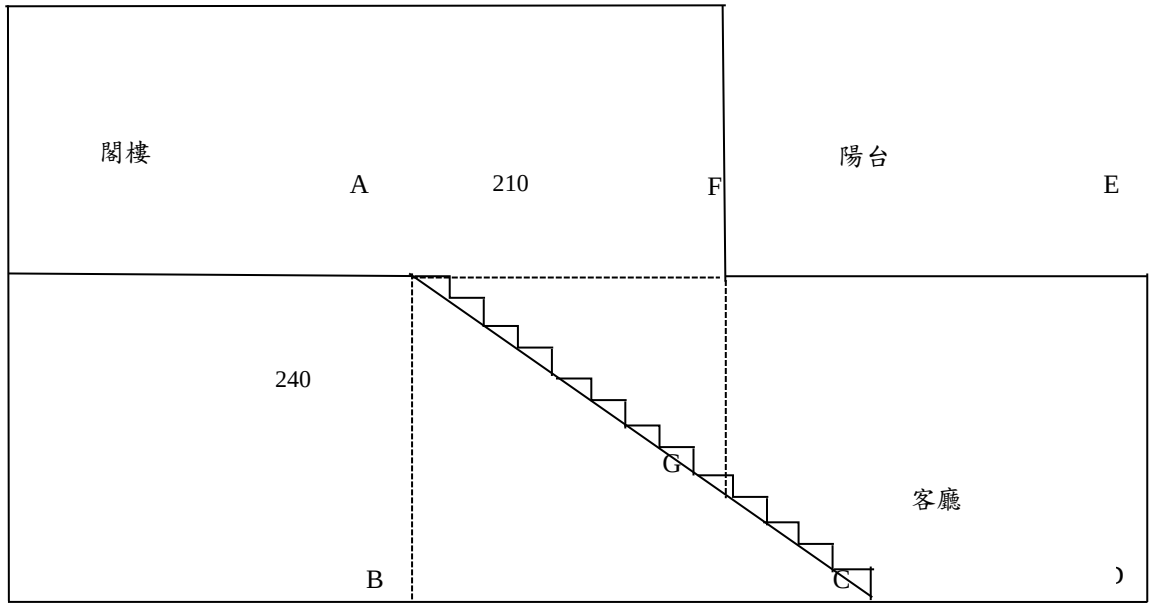
當馬桶沖水之後，會以40cm3/秒的速率注水入馬桶內，下圖表示當馬桶沖水完畢後，開始注水時馬桶水面高度H(cm)對時間*t*(秒)的關係圖：



The Blueprint

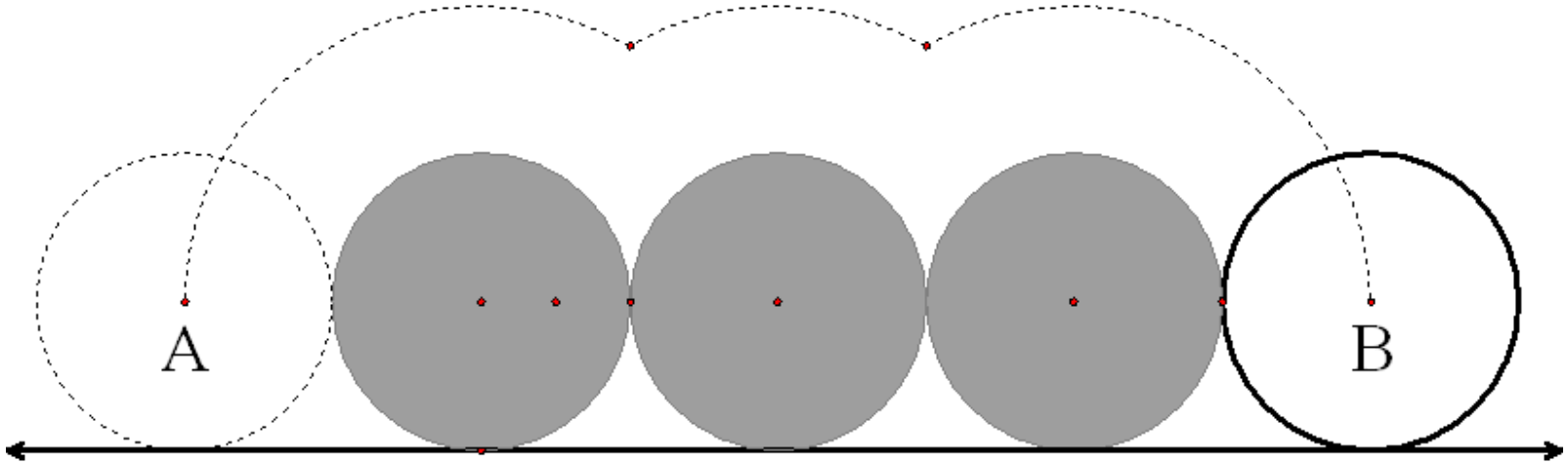
下圖為樓中樓的剖面圖，在建造客廳到閣樓的樓梯 時，實際測量施工現場得到以下數據：

客廳高240cm，樓梯洞口寬210cm，閣樓陽台寬250cm。



The Rolling Coin

有若干個半徑為1的硬幣(皆為半徑相同的等圓)按下圖的形式固定排列著，
有一個半徑也是1的 硬幣沿著這些等圓的上方無滑動地滾動到圓 的位置。



Higher Degree, Higher Income

台灣社會對這20年來的教改正反評價不一，而其中一項教改主張就是「廣設大學」。贊成者認為大學錄取率太低，形成文憑重於能力的社會現象，所以應該要改革；反對者認為廣設大學會造成文憑貶值，大學生會薪資下降，例如：很多大學生抱怨現在大學畢業後的起薪是22K（K表示千元、所以22K表示22000元）。

以下是根據政府統計資料所列出的表格：

應屆畢業生平均月薪	1999年		2006年	
	人數（萬人）	平均月薪（K、千元）	人數（萬人）	平均月薪（K、千元）
專科學歷	12	21	6	20
高職學歷	16	24	10	23
大學學歷	9	27	22	26

Austinogebia edulis (Mud-Shrimp)

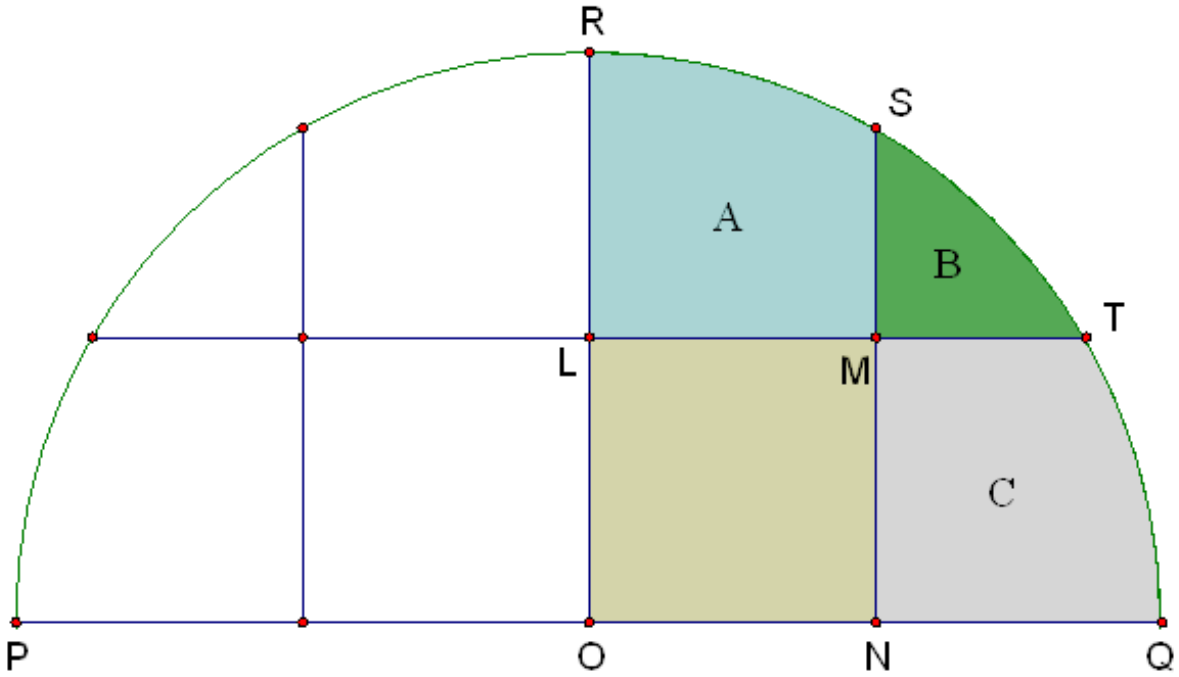
蝲蛄蝦是生存在沙灘上的一種生物，透過觀察發現一隻**蝲蛄蝦**會在沙灘上挖出兩個大概原子筆寬度的洞穴，一個當入水口，另一個當出水口，讓它過濾食用海水中的小生物。

一般估算沙灘上洞穴數的方法如下：我們利用一個邊長50公分的正方形木框(如右圖)，隨機抽取沙灘上若干位置放置木框，去計算這些沙灘區域洞穴數量的平均值，當作這沙灘上每所含的洞穴數量的標準值，就可以推估整個沙灘的洞穴數量。最後再將估算出的洞穴總數量除以2就是該沙灘區域內**蝲蛄蝦**數量。

木框

Semicircle Glass Cutting

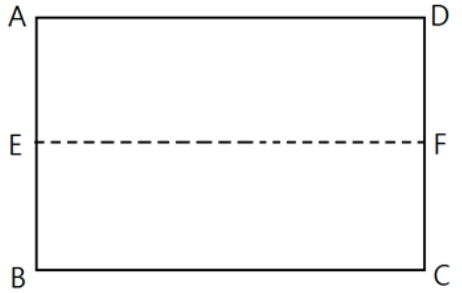
現有一片直徑120公分的半圓形玻璃，欲將半圓分割成如下圖的八塊，其中 為直徑， 為垂直於 的半徑，切割線為 及分別過半徑 中點的垂直線。本問題的圓周率 以3.14計算即可 ，最後答案請四捨五入至小數第二位。



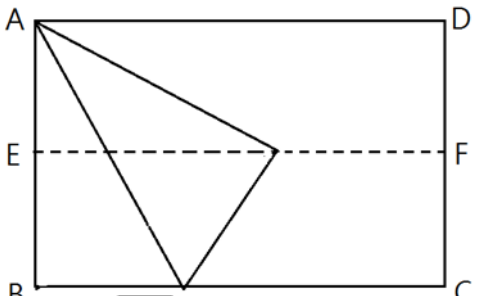
Paper Folding

芊芊將長方形紙片ABCD的長邊$\overline{BC}$邊沿$\overline{AD}$邊對折，得折痕$\overline{EF}$，攤開長方形ABCD後，再將$\overline{AB}$邊往上折，直至B點落在折痕$\overline{EF}$上(如圖所示)

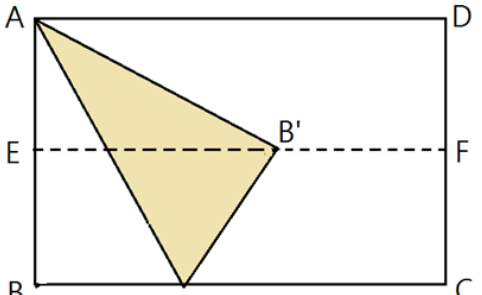
(圖一)



(圖二)



(圖三)



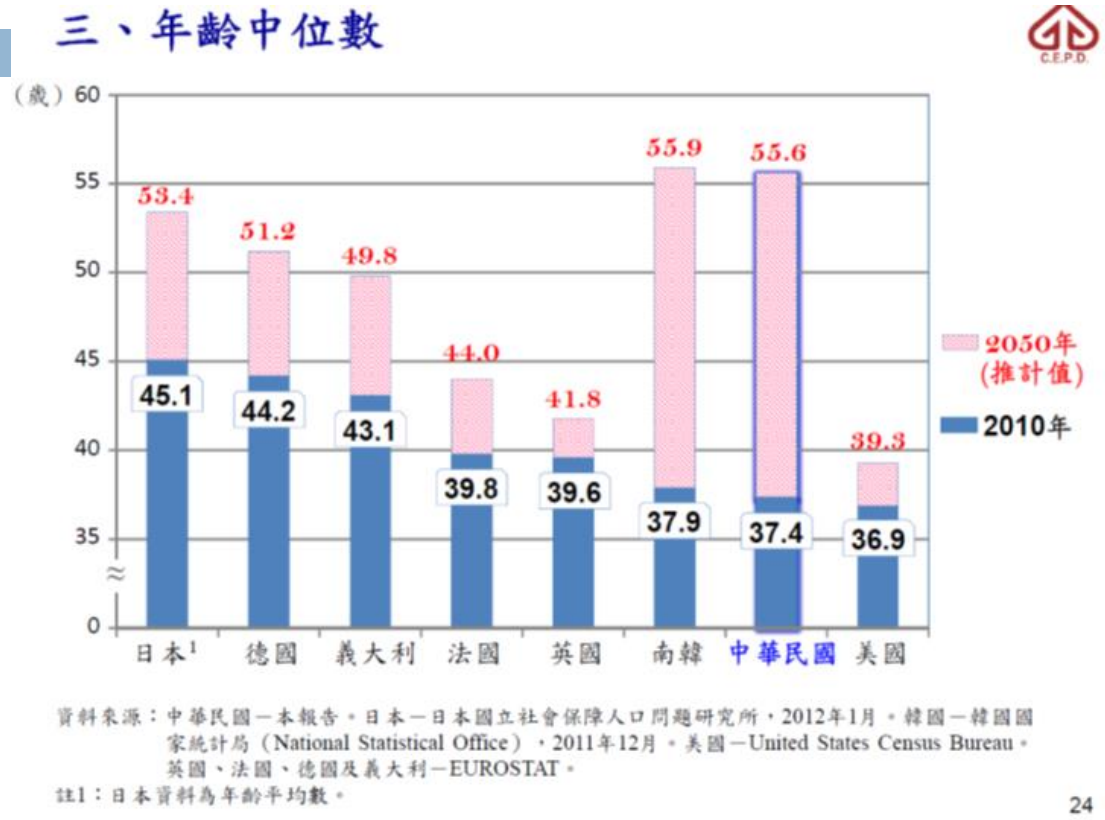
(圖四)

The Median of Age among Citizens-1

行政院經濟建設委員會於2012年8月的人口推計報告指出，年齡中位數，可用來代表整個人口的年齡水平，如下圖。目前(2010)台灣的中年人（年齡中位數）大約37.4歲，到2050年人口年齡中位數將是55.6歲，意味著即使到了56歲，在當時的台灣仍算是青年人。

國際上更將65歲以上人口占總人口比率達到7%、14%及20%，分別稱為高齡化社會、高齡社會及超高齡社會。台灣已於1993年成為高齡化社會，預計將於2018年及2025年分別邁入高齡社會及超高齡社會。

The Median of Age among Citizens-2



而年齡中位數的計算方式如下：

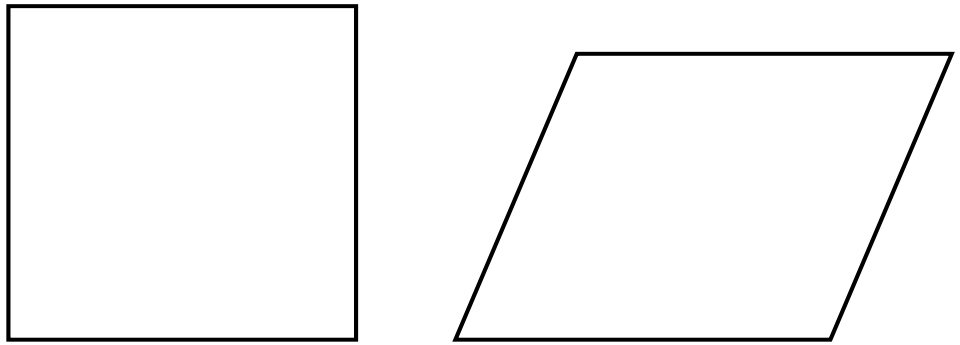
$$\text{中位數組的年齡下限值}+\left\{\frac{\frac{\text{人口總數}}{2}-\text{中位數組之前各組人數累計}}{\text{中位數組的人口數}}\right\}\times\text{組距}$$

Earthquake

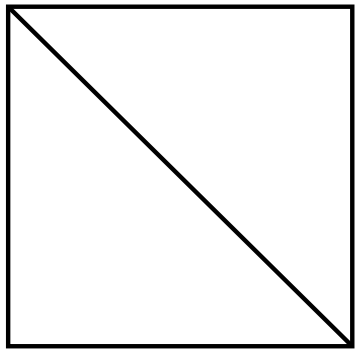
地震是地殼快速釋放能量的過程中所造成的震動。地震發生時，震動的發源處稱為「震源」，至於所謂的「震央」，就是震源在地表的投影點(即地球球心、震源之連線與地表的交點)，而震源與震央的距離，就稱作「震源深度」。根據震源深度，我們將地震分為四類：極淺地震(深度在0-30公里)、淺層地震(深度在30-70公里)、中層地震(深度在70-300公里)和深層地震(深度在300-700公里)。假設地球是一個完美的球形，試回答下列的問題：(紅字部分是否需刪除?)

Fixation on the Billboard

廣告看板通常是一個多邊形的邊框貼上板面所組成，邊框如果沒有固定好，在風吹雨打下就會變形，例如圖一左邊的正方形邊框很容易變形為平行四邊形。一種解決方式是在看板背後加上一些連接頂點的支架，例如圖二就是在正方形對角線加上一根支架來固定邊框。



圖一



圖二

The Squareness of the Quadrilateral

我們知道正方形是矩形，但是矩形不一定是正方形，小安想要設計一個公式來量測矩形與正方形的接近程度，稱之為**方形度**。

假設矩形的相鄰兩邊長為 $a,b(a\geq b)$，小安設計了 A 和 B 兩種公式定義矩形的方形度：

矩形的方形度 A = $a-b$

矩形的方形度 B= $\frac{a}{b}$