

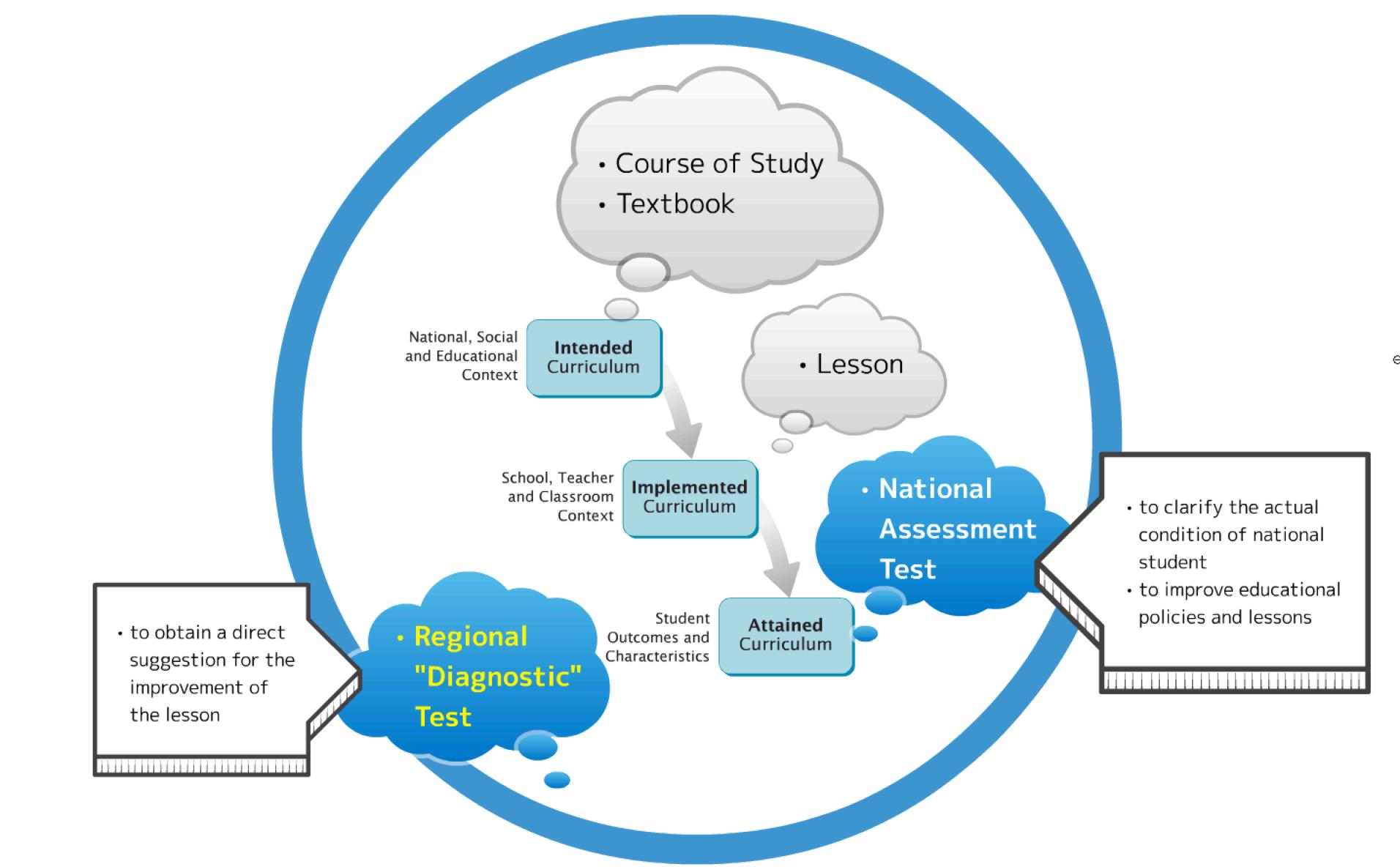
OPEN ENVIRONMENT FOR WORLDWIDE MATHEMATICAL EDUCATION

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Japanese Elementary School Students Math Performance: A Case of Tottori Prefecture regional "Math Diagnostic Test"

Tatsuya Mizoguchi Tottori University, Japan

Yusuke Shinno Osaka Kyoiku University, Japan





Tottori Prefecture



島根県

Western



Central



Eastern



広島県

岡山県



Tottori Elementary Math Diagnostic Test

a suitable opportunity for teachers to study mathematical and pedagogical content knowledge and recognize what are emphasis/important contents through the curriculum:

- What kinds of problems are essential/appropriate to verify students' math performance?
- What kind of performance do students show to such problems?

Procedure

- 1) determination of the Test members in math subcommittee of the district
- 2) determination of the grade staffs to analyze past results and discuss improvements of the year
 - 3) improving such problems and develop new problems
 - 4) planning the Test (as a whole)
 - 5) workshop for the planning tests
 - 6) ordering to print the Test booklets
 - 7) implementation of the Test
 - 8) scoring of the Test (by homeroom teachers of each school)
 - 9) each school reports results (scores) of the Test
- 10) staffs aggregate and analyze the results, and make an annual report

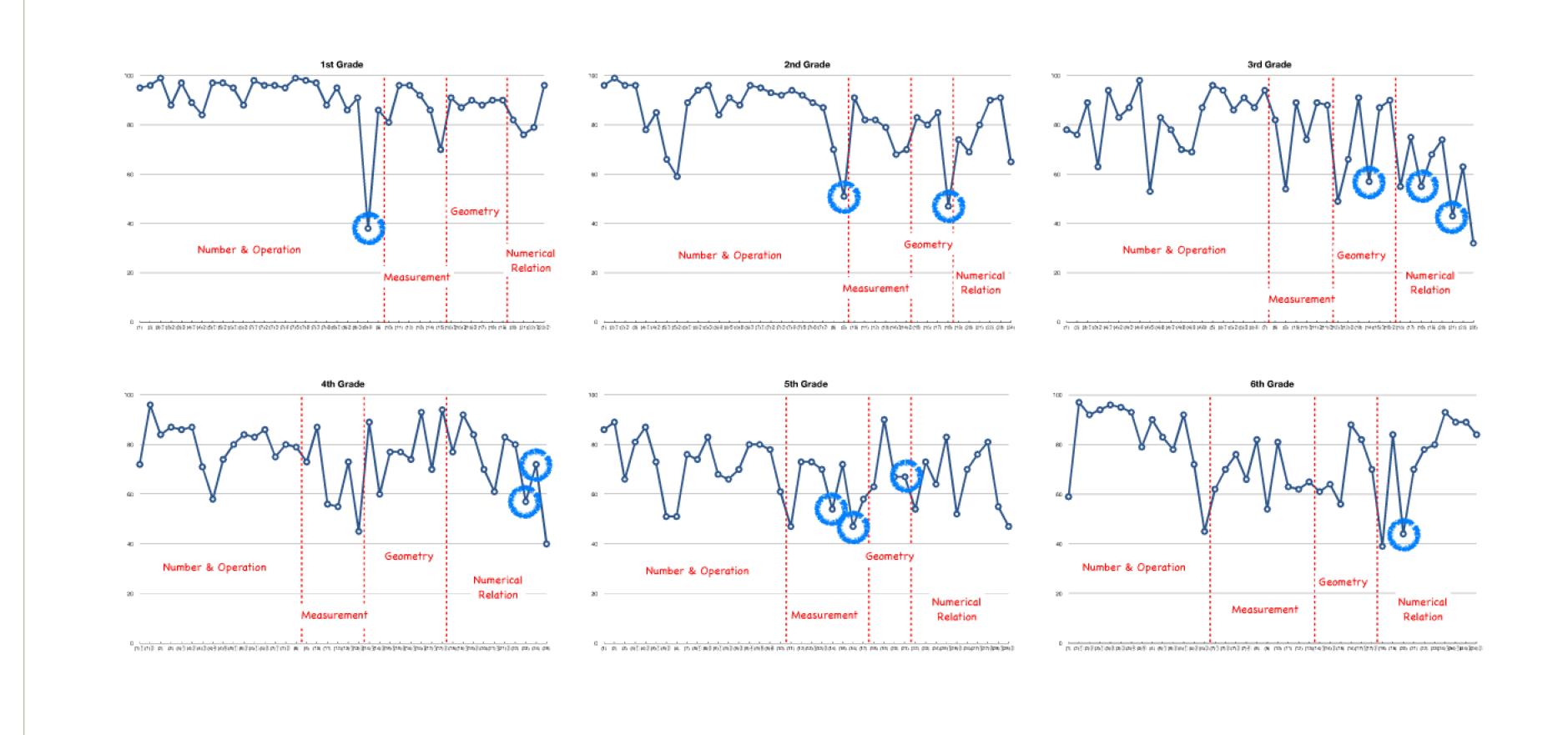
Examinee students

Table 1. Number of students by each grade							
Grade	1 st	2 nd	3 rd	4 th	5 th	6 th	
Number of students	4908	4867	4968	5138	5057	5241	

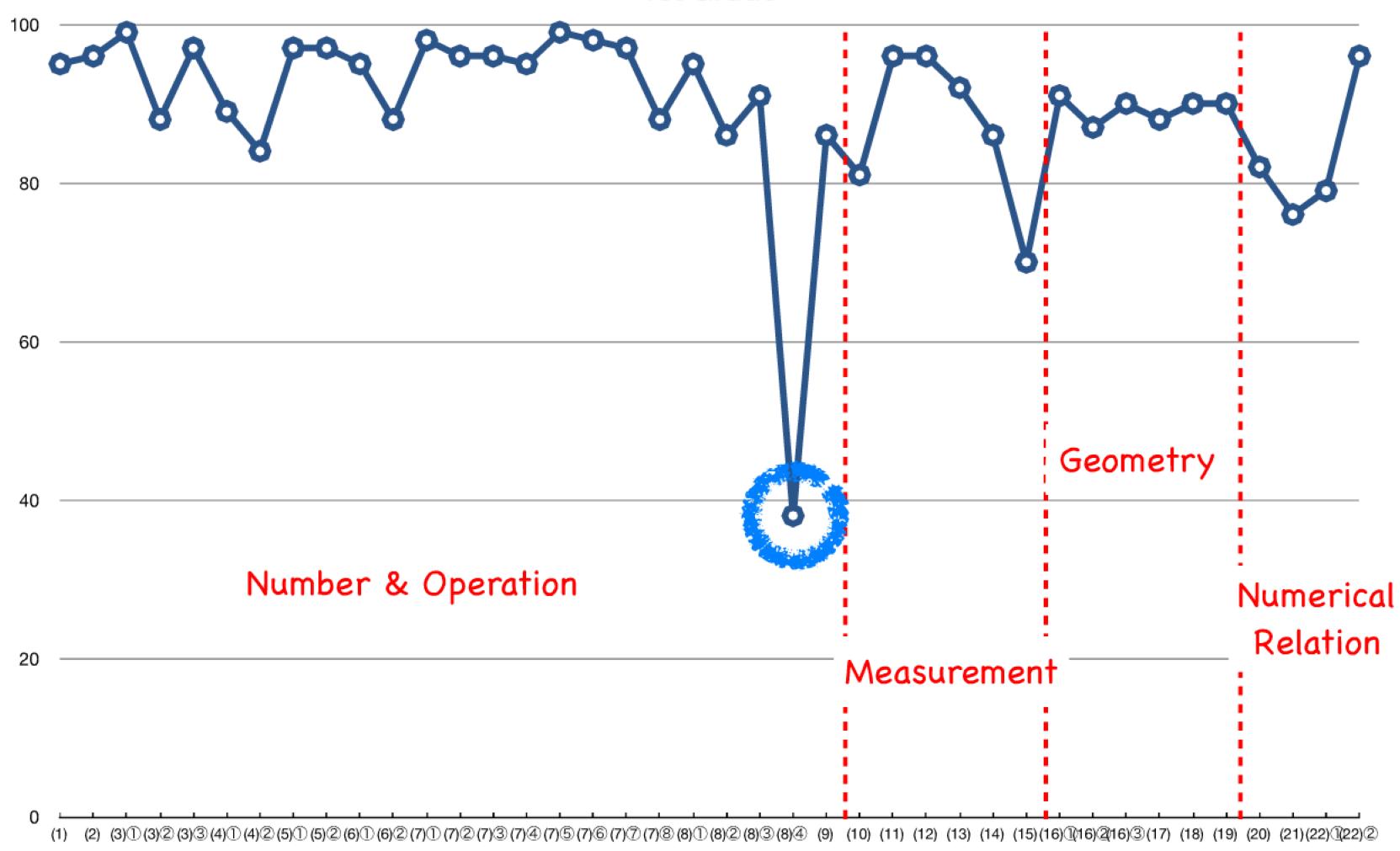
Table 2. National Assessment Test 2013							
Tottori Prefecture (%) National Average(%)							
Elementary Math A	78.1	77.3					
Elementary Math B	60.2	58.6					

Problems for examining the thinking process

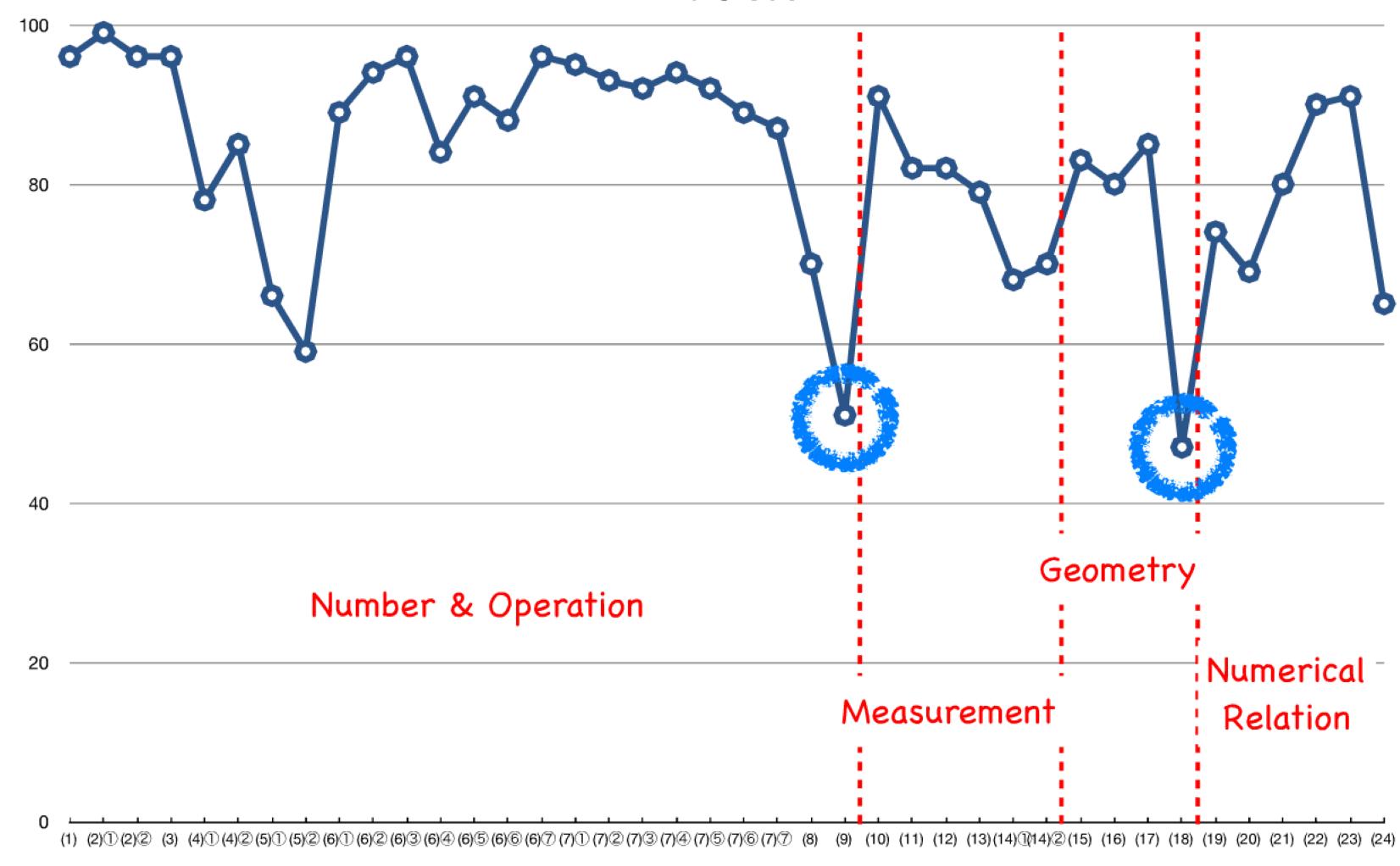
Although each Test problem requires only the answer basically, "problems for examining the thinking process" require describing the thinking process by an indication: "Let's leave your writing/trajectory of thought."



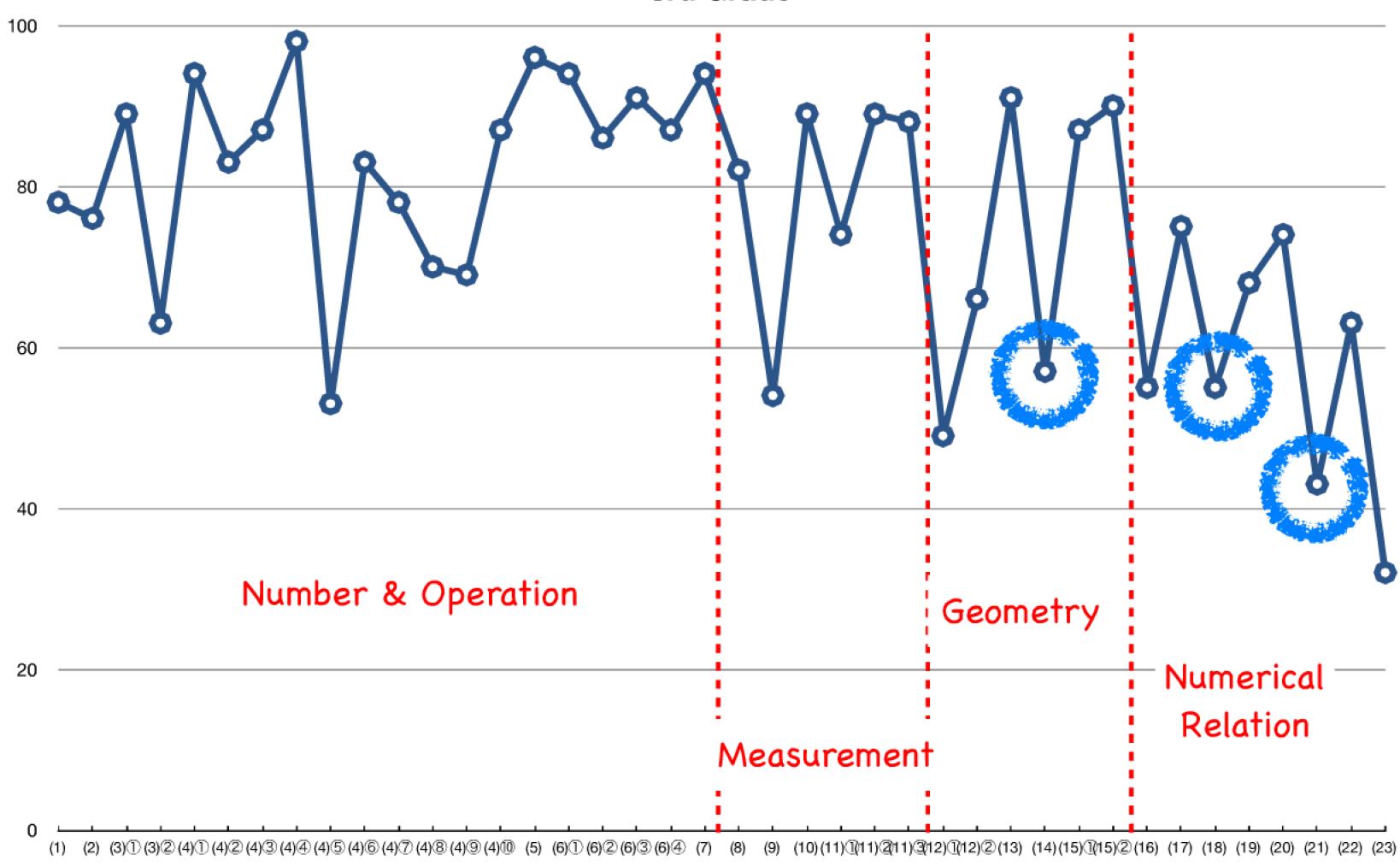




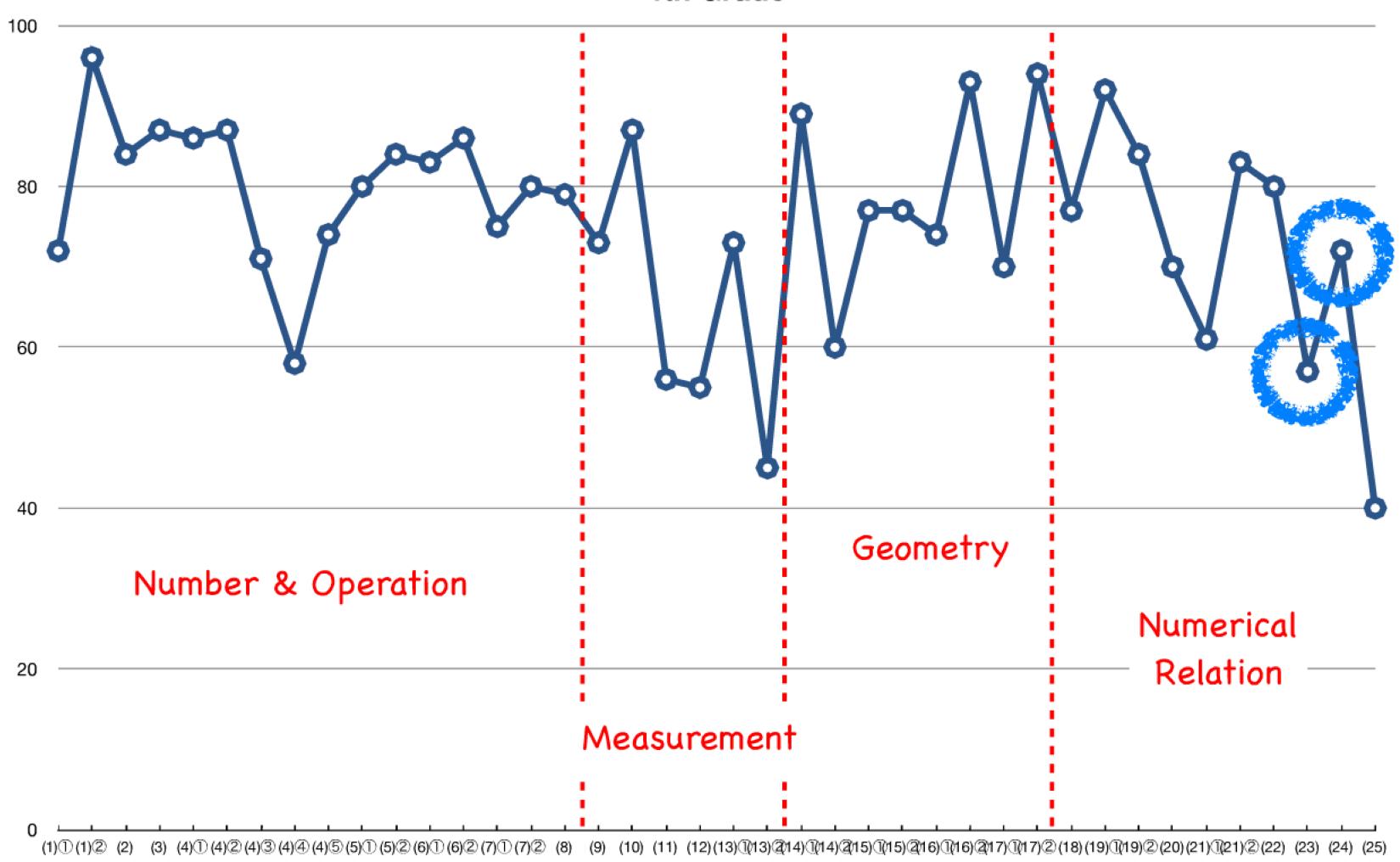


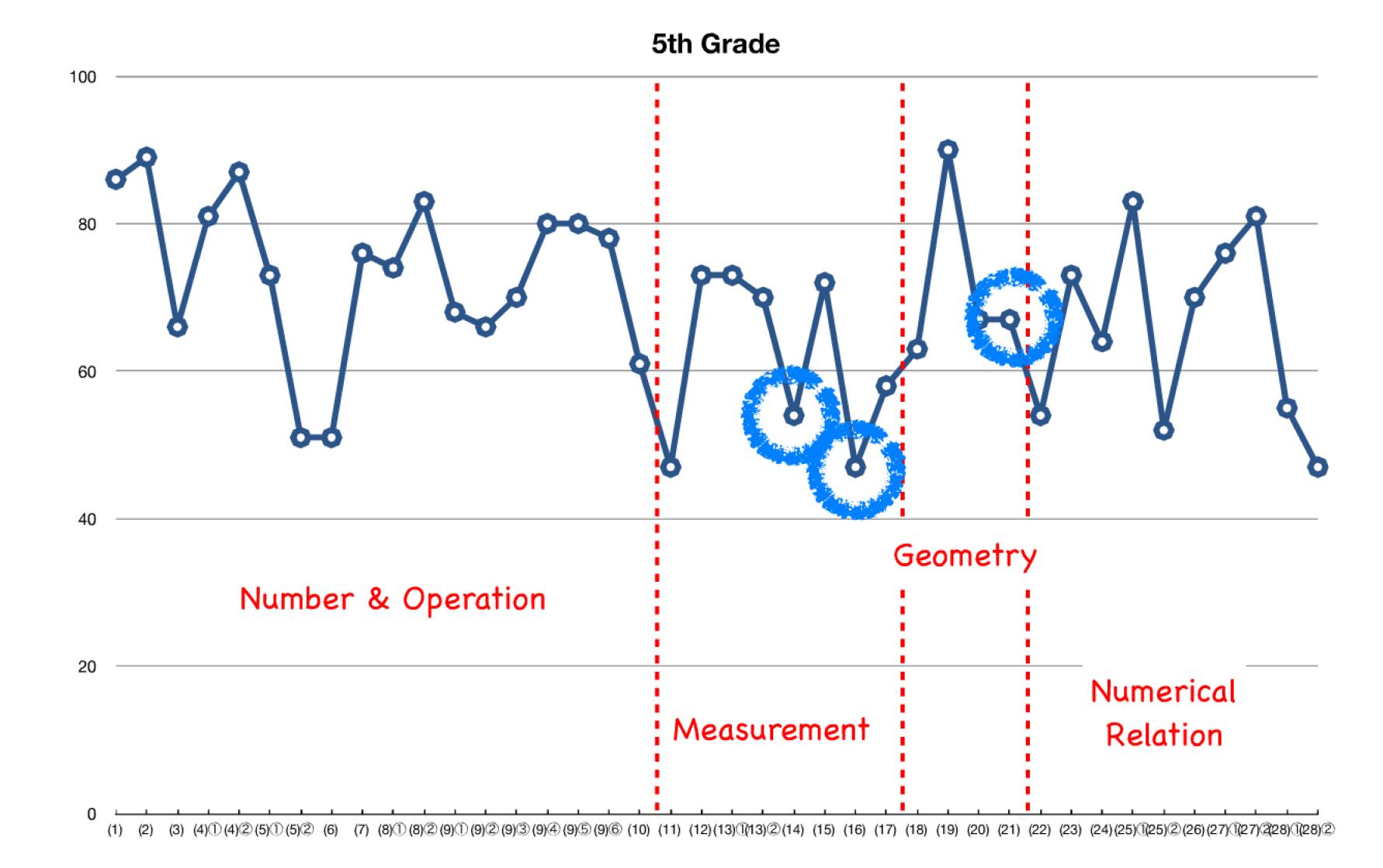


3rd Grade

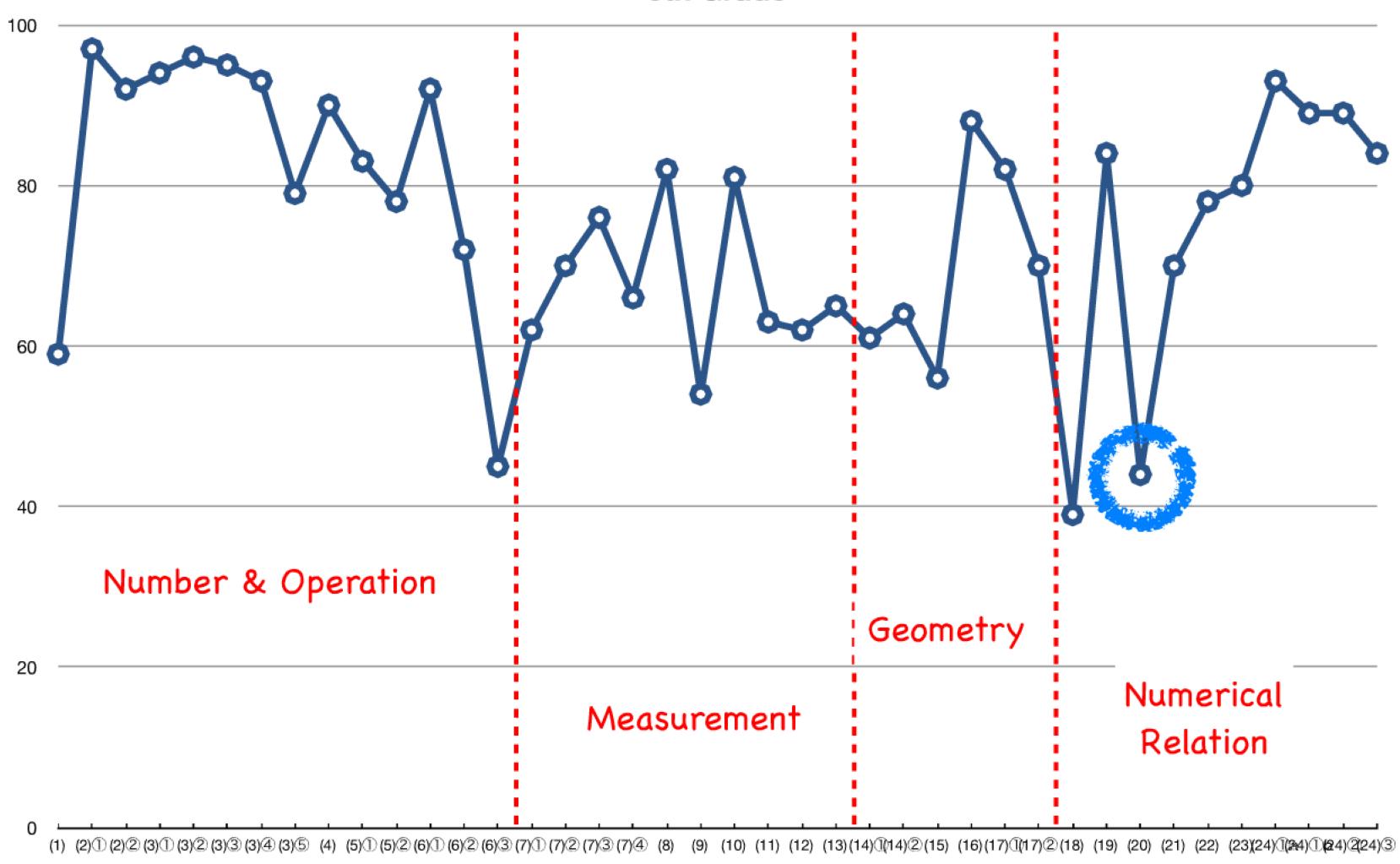








6th Grade



Problem of 1st Grade

Prob.	Problem	Ave. of c	orrect ans	Errors	
No.	rroblem	2012	2013	2014	(Rate to the whole errors)
(8) ④	When distributing 12 pencils by one for one person, 5 pencils are left. When 10 pencils, how many pencils are left?	39	39	38	1. 7 (29%) 2. 5 (29%) 3. 2 (14%) 4. others (14%) 5. no answer (14%)

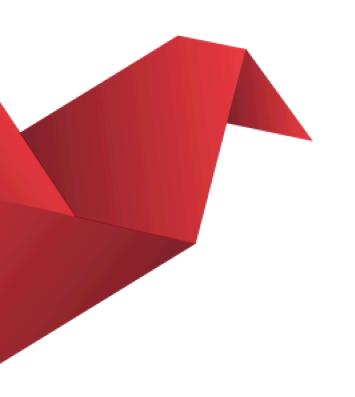
Intention of the Problem

- · To find the unknown number from the given condition;
- · To identify the number of pencils and persons;
- To represent a problem situation with a diagram or a picture.

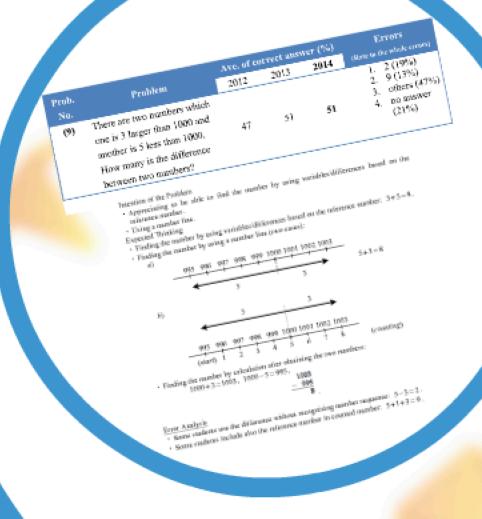
Expected Thinking

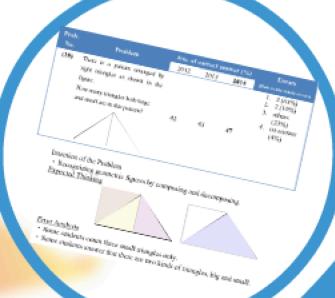
- Finding the distributed number of the pencils by subtracting the number of remainders from the number of the beginning. This number is the number of people. Then, subtracting the number of pencils to distribute to seven persons from ten: 12-5=7, 10-7=3.
- Finding the difference of the number of pencils between the first and the second situation. Since the two small pencil in the second situation, the number of remaining pencils also become two small: 12-10=2, 5-2=3

- Some students can't do two-step thinking, they finish problem solving by finding the number of distributed pencils: 12-5=7.
- Some students subtract five remainders from ten pencils: 10-5=5.
- Some students misread the problem as "how many pencils remain when distributing 10 from 12?": 12-10=2.



Problems of 2nd Grade





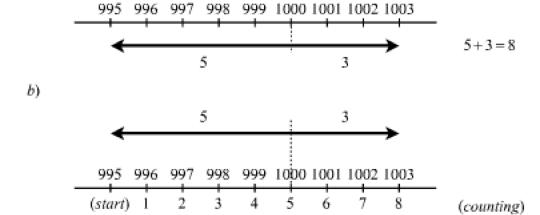
Prob.	Problem	Ave. of o	correct ans	Errors	
No.	robiciii	2012	2013	2014	(Rate to the whole errors)
(9)	There are two numbers which one is 3 larger than 1000 and another is 5 less than 1000. How many is the difference between two numbers?	47	51	51	1. 2 (19%) 2. 9 (13%) 3. others (47%) 4. no answer (21%)

- Appreciating to be able to find the number by using variables/differences based on the reference number.
- · Using a number line.

Expected Thinking

- Finding the number by using variables/differences based on the reference number: 3+5=8.
- · Finding the number by using a number line (two cases):

a)



• Finding the number by calculation after obtaining the two numbers:

Error Analysis

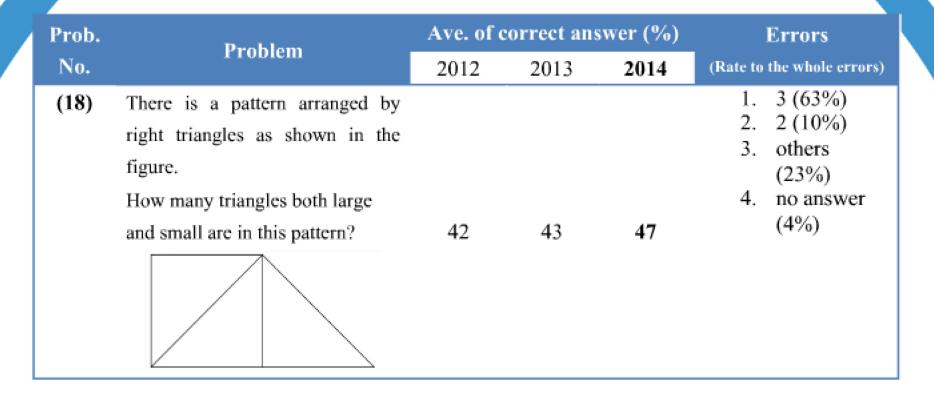
- Some students use the difference without recognizing number sequence: 5-3=2.
- Some students include also the reference number in counted number: 5+1+3=9.

Prob.
No.
(18)

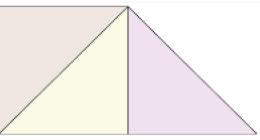
Problem

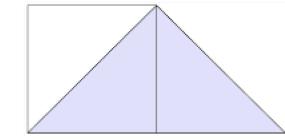
There is a pattern arranged by 2012 correct
figure.

How many triangles both large
and small are in this pattern?



Recognizing geometric figures by composing and decomposing.
 Expected Thinking





- · Some students count three small triangles only.
- · Some students answer that there are two kinds of triangles, big and small.

Problems of 3rd Grade

1	7		Ave. of e	orrect answ	er (%) 2014	(Rate to the whole except)	
1	Prob. No.	Problem Balls are packed in a box	2012	2013		1. 5 cm (46%) 2. 9 cm (13%) 3. 2 cm (13%) 4. others	
	(14)	Balls are passed in the accumulaty as shown in the figure. How long (cm) is the radius of a ball!	54	54	57	(23%) 5. 10 answer (5%)	
						o of vertical in	

Interestical of the Problem

Outdoorwanding spherical discuster and radius in relation to the length of vertical and horizontal of the box.

Expected Vinking.
Finding the radius after obtaining the diameter. 18+3=6, 6+2=3 3 cm.
Finding the radius of the basis of the number of radii (per side): 2×3=6, 18+6=3 3 cm.
Find the radius on the basis of the number of radii (per side): 1×3=6, 18+6=3 1 cm.

Expected Vinking.

Some suddents answer the diameter: 18+3=6, 6 cm.

Some suddents recognize incorrectly the length of one side of the box as a diameter.

Some suddents recognize incorrectly the length of one side of the box in 18+9=2 (cm).

. Some students divide the length of one side of the box by nine balls: $18+9=2\,(cm)$.



7	٥			Am. of o	personal area	2014	Majorial Service State (1994)
/	7	Prob-	Problem	3617	3013		1. 2 (24%) 2. 28 (7%) 3. 16 (3%)
1		(21)	Trace we tined in a new sweey by 2m. These of both cuts one every	35	19	45	4. others (87%)
l			sters. More money trace and fined?				



1	Prob.	Problem	Ave. of o	correct ans	Errors	
	No.	Froblem	2012	2013	2014	(Rate to the whole errors)
	(14)	Balls are packed in a box accurately as shown in the figure. How long (cm) is the radius of a ball?	54	54	57	1. 6 cm (46%) 2. 9 cm (13%) 3. 2 cm (13%) 4. others (23%) 5. no answer (5%)

 Understanding spherical diameter and radius in relation to the length of vertical and horizontal of the box.

Expected Thinking

- Finding the radius after obtaining the diameter: $18 \div 3 = 6$, $6 \div 2 = 3$ 3 cm.
- Find the radius on the basis of the number of radii (per side): $2 \times 3 = 6$, $18 \div 6 = 3$ 3 cm.

Error Analysis

- Some students answer the diameter: $18 \div 3 = 6$, $\underline{6}$ cm.
- Some students recognize incorrectly the length of one side of the box as a diameter:

$$18 \div 2 = 9 \text{ (cm)}.$$

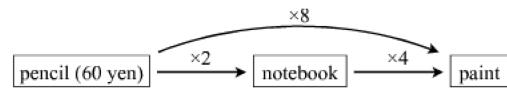
• Some students divide the length of one side of the box by nine balls: 18 + 9 = 2 (cm).

Prob.	Problem	Ave. of correct answer (%)			Errors	
No.	Froblem	2012	2013	2014	(Rate to	the whole errors)
(18)	There are pencils, notebooks, and paints. A pencil is 60 yen. A notebook is twice the price of the pencil. A paint is four times the price	57	57	55	1. 2. 3. 4.	6 times (44%) 480 times (10%) 2 times (10%) 4 times (5%)
	of the notebook. How many times the paint is the price of the pencil?				5. 6.	others (26%) no answer (5%)

· Solving a problem by using multiplication operators.

Expected Thinking

• Finding the number by calculating operators: $2 \times 4 = 8$ 8 times.



• Comparing the prices of pencil and paint after finding the prices of notebook and paint in turn: $60 \times 2 = 120$, $120 \times 4 = 480$, $60 \times \square = 480$, 8 is appropriate for \square ; or $480 \div 60 = 8$.

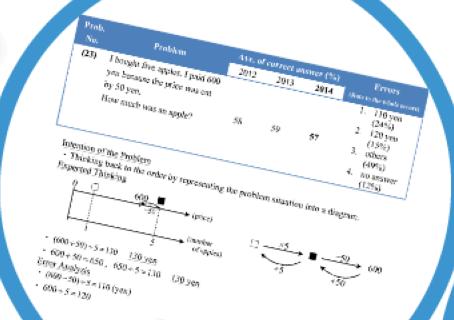
- Some students add the multiplication operators: 2+4=6 (times).
- · Some students obtain the price of paint (their thinking process are correct).
- · Some students answer either of operators.

10 · 2 –) (cm).

a side of the how burning haller 18 . 0 - 2 (am)



Problems of 4th Grade



			Ave. of c	potrect ons	wer (%)	English due to the whole corners
4	Prob.	Problem	2012	2013	2014	1. 72 kg
	No. (24)	The morkey weight 12 kg, and is three times the weight of the rabbit. The weight of the rabbit is twice the weight of the squimel. How many kg does the squimel	72	74	72	(25%) 2. 4 kg (12%) 3. others (38%) 4. no answer (25%)
		weigh?				

Solving a problem by using operators with drawing diagrams.
 Expected Thinking

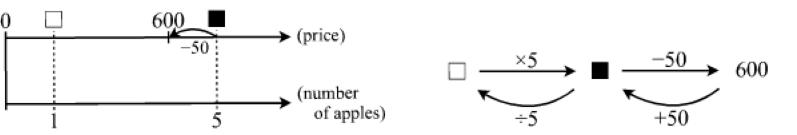


- 12+3=4, 4+2=2 2 kg $12+(2\times 3)=2$ 2 kg

12+(2×3)=0 a.58
 Error Analysis
 Some students can't represent the problem situation into a diagram or a figure, and consequently they do incorrect calculation variously with using the numerical values of the problem sentence: 12×3×2=72, 12+3=4, 12-3-2=7.

1	Prob.	Problem	Ave. of	correct ans	Errors	
	No.	1 robiem	2012	2013	2014	(Rate to the whole errors)
	(23)	I bought five apples. I paid 600 yen because the price was cut				1. 110 yen (24%)
		by 50 yen.	58	50	57	2. 120 yen (15%)
		How much was an apple?	26	59	31	3. others (49%)
						4. no answer (12%)

· Thinking back to the order by representing the problem situation into a diagram. **Expected Thinking**



•
$$(600+50) \div 5 = 130$$
 130 yen

•
$$600 + 50 = 650$$
, $650 \div 5 = 130$ 130 yen

Error Analysis

•
$$(600-50) \div 5 = 110 \text{ (yen)}$$

•
$$600 \div 5 = 120$$

Problem

Prob. No.

(24)

The monkey weig is three times the rabbit. The weig is twice the

squirrel. How many k

weigh?

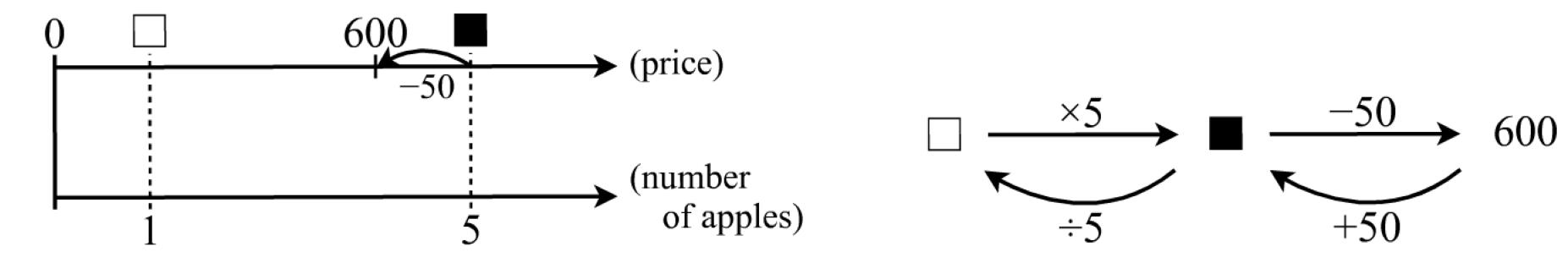
Intention of the Pr · Solving a prob Expected Thinki

monkey rabbit

. 12+3 . 12÷(Error A

• Thinking back to the order by representing the problem situation into a diagram.

Expected Thinking



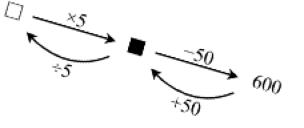
•
$$(600+50) \div 5 = 130$$
 130 yen

•
$$600 + 50 = 650$$
, $650 \div 5 = 130$ 130 yen

•
$$(600-50) \div 5 = 110 \text{ (yen)}$$

•
$$600 \div 5 = 120$$

^{roblem} situation into a diagram.



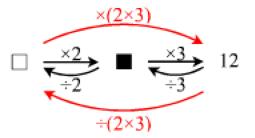
Prob.	Problem	Ave. of	correct ans	Errors	
No.	Problem	2012	2013	2014	(Rate to the whole errors)
(24)	The monkey weighs 12 kg, and is three times the weight of the rabbit. The weight of the rabbit is twice the weight of the squirrel. How many kg does the squirrel weigh?	72	74	72	1. 72 kg (25%) 2. 4 kg (12%) 3. others (38%) 4. no answer (25%)

Intention of the Problem

· Solving a problem by using operators with drawing diagrams.

Expected Thinking





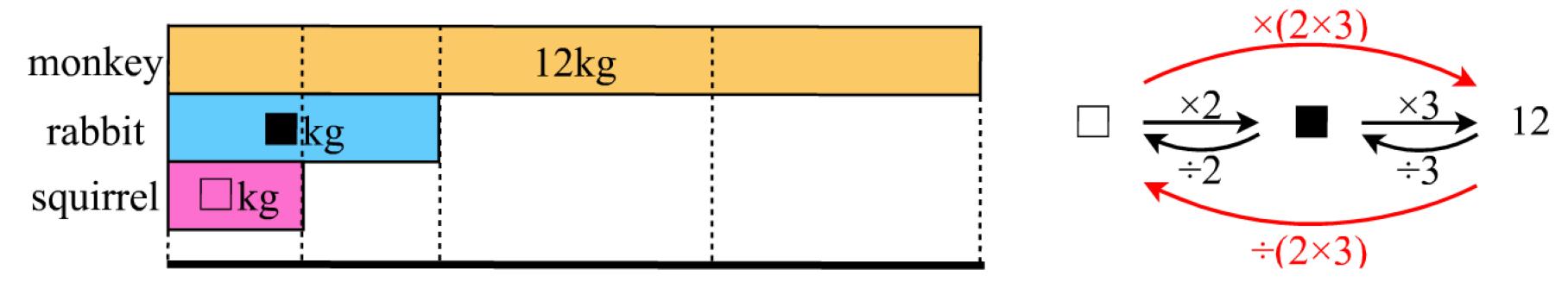
- $12 \div 3 = 4$, $4 \div 2 = 2$ 2 kg
- $12 \div (2 \times 3) = 2$ 2 kg

Error Analysis

• Some students can't represent the problem situation into a diagram or a figure, and consequently they do incorrect calculation variously with using the numerical values of the problem sentence: $12 \times 3 \times 2 = 72$, $12 \div 3 = 4$, 12 - 3 - 2 = 7.

Solving a problem by using operators with drawing diagrams.

Expected Thinking



•
$$12 \div 3 = 4$$
, $4 \div 2 = 2$ 2 kg

•
$$12 \div (2 \times 3) = 2$$
 2 kg

Error Analysis

• Some students can't represent the problem situation into a diagram or a figure, and consequently they do incorrect calculation variously with using the numerical values of the problem sentence: $12 \times 3 \times 2 = 72$, $12 \div 3 = 4$, 12 - 3 - 2 = 7.

Problems of 5th Grade

					Errors
			erack aroma	ar (*Side	(Male table whole course)
			2017	361.4	and the second
Trob	Peablem	3012	2013		43.7% 9
Ne					n 32.00°
					44.75% A
(14	Contract I trad take on the				3. 40 m² (3%)
	whole flower bade.	44	58	94	4. oben (31%)
- 1	20.18	55			
	April 84				5. 80 (18%)
	2 A FR				
	gasahed *				
	- W W				township of a
	Interfere of the Publish Finding a way of delations the		or tanks 10	y using #	DESCRIPTION OF THE PERSON OF T
la de	Intersion of the Publish	years of the	W-SENSESSEE AND ADDRESS OF THE PERSON NAMED IN COLUMN TWO IS NOT THE PERSON NAMED IN COLUMN TO T	dan.	
	Confess is want of discoverable of	q editeriore		and the second	ar:

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Finding a way of obtaining the soon of plants seem.

Finding a way of obtaining the soon of plants seem.

Finding a way of obtaining the soon of plants and explantation of the which garden:

Executed Thirties.

Substructing the soon of possesse park from the soon of the which garden:

(a+1+1)(a+1+2=2, 23, a).

(2+1+5)(a+1+2=2, 23, a).

(2+1+5)(a+1+2=2, 23, a).

Considering forward and (costs to be soon temperature: (2+4)(a+1+2+1)(a+1+2=2, a).

(a+1)(a+1+2=2, a).

(a+1)(a+1+2=2, a).

Ever Analysis.

Some students calculate the mean by using the "negetherane" of a perallelogram as the "neight": (5+2)×5-35.

Some students does represent the area of passage mat: (6+1+1)×4-32.

Some students does represent the area of passage mat: (6+1+1)×4-2-16, text2-13.

Some students consistent that there are pre-some temperature of a parallelogram as the "neight" and by consistent in mean both by using the "hyperstrate" of a parallelogram as the "neight" and by consistent in these are two some traperature.

Pro						
No.		Problem	Ave			
(16)	Laro hay out	en five	2012	correct at	15Wer (%)	
- /	THE PERSON NAMED OF THE PE	. Six		2013	200	Eller
	fifth score	outh was 88. The				(Rate to the whole crews)
	fifth score wa average of all	s 98. Find the	40	42		1. 93 (34%) 2. 37.2 (6%) 3. 130 (6%)
Intention		Tive exam.			47	128 (6%)
Findin	g overall aver	by using the temp				4. others (33%)
Expected Finding	Thinking areing	e by using the tem-	Vira-			DO STISTURE
adding t	overall average the fifts	by colons	wary avera	ge.		(21%)

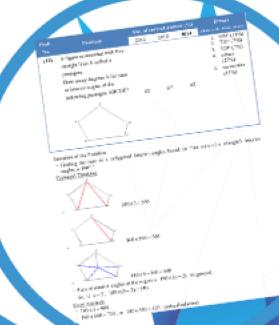
Appected Thinking

Finding overall average by calculating the total from temporary average to the fourth and adding the fifth score; $88 \times 4 = 352$, 352 + 98 = 450, 450 + 5 = 90; or $(88 \times 4 + 98) + 5 = 90$,

Finding constant answers the distribution the distribution that the state of the fourth and the state of the

Finding overall average by dividing the difference between the temporary average to the Error Analysis $88 \times 4 = 352, \quad 352 + 98 = 450, \quad 450 + 5 = 90; \text{ or } (88 \times 4 + 98) + 5 = 90.$ Fourth and the fifth score into five equal parts: $98 - 88 = 10, \quad 10 + 5 = 2, \quad 88 + 2 = 90; \text{ or } (98 - 88) + 5 = 2, \quad 88 + 2 = 90.$

Some students consider the total of five times to be five times of 98 (average): $88 \times 4 = 352, \quad 98 \times 5 = 490, \quad 490 - 352 = 138.$



Prob.	Duoblam	Ave. of	correct ans	Errors	
No.	Problem	2012	2013	2014	(Rate to the whole errors)
(14)	There is a parallelogram-like garden. Find the area of the whole flower beds. 2m 1m 5m flower bed 5m flower bed 5m flower bed 6m 1m 1m	55	58	54	1. 35 m ² (37%) 2. 32 m ² (11%) 3. 40 m ² (3%) 4. others (31%) 5. no answer (18%)

· Finding a way of obtaining the area of flower beds by using quadrature formulas of a parallelogram and a trapezoid, and equivalency transformation.

Expected Thinking

· Subtracting the area of passage part from the area of the whole garden:

$$(6+1+1)\times 4-1\times 4=28$$
 28 m^2 ,
 $(2+1+5)\times 4-1\times 4=28$ 28 m^2 .

- Considering flower bed parts to be two trapezoids: $(2+6)\times 4 \div 2 + (5+1)\times 4 \div 2 = 28 \text{ (m}^2)$.
- · Finding the area as a parallelogram by moving a part of the flower bed:

$$(6+1)\times 4 = 28 \text{ (m}^2), (2+5)\times 4 = 28 \text{ (m}^2).$$

Error Analysis

- · Some students calculate the area by using the "hypotenuse" of a parallelogram as the "height": $(5+2) \times 5 = 35$.
- Some students don't remove the area of passage part: $(6+1+1)\times 4=32$.
- Some students consider that there are two same trapezoids: $(2+6)\times 4 \div 2 = 16$, $16\times 2 = 32$.
- · Some students calculate the area both by using the "hypotenuse" of a parallelogram as the "height" and by considering that there are two same trapezoids: $(6+1+1)\times 5=40$.

 $N_{0.}$ (16)

av Intention of the

· Finding ove Expected Think · Finding overa

adding the fiftl

· F_{Inding} overall fourth and the fift

Error Analysis 98-88=

 $(88+98) \div 2 = 93.$ $(88+98) \div 5 = 37.2$ Some students consisted

ems or

Errors
Errors te to the whole errors)
1. 35 m ² 1. (37%)
2 32 m
10 m2 (3 %)
1 others
5. (31%) 5. no answer (18%)
(18/4)
quadrature formulas of a
quadrature

Pro	ob.	Problem	Ave. of correct answer (%)			Errors
N	0.	Frontein	2012	2013	2014	(Rate to the whole errors)
(10	6)	Taro has taken five examinations. The average of exam up to fourth was 88. The fifth score was 98. Find the average of all five exam.	40	42	47	1. 93 (34%) 2. 37.2 (6%) 3. 138 (6%) 4. others (33%) 5. no answer (21%)

Intention of the Problem

· Finding overall average by using the temporary average.

Expected Thinking

trop on the Finding overall everage by calculating the total from temporary average to the fourth and

fifth score:

$$88 \times 4 = 352$$
, $352 + 98 = 450$, $450 \div 5 = 90$; or $(88 \times 4 + 98) \div 5 = 90$.

verall average by dividing the difference between the temporary average to the the fifth score into five equal parts:

$$8-88=10$$
, $10 \div 5 = 2$, $88+2=90$; or $(98-88) \div 5 = 2$, $88+2=90$.

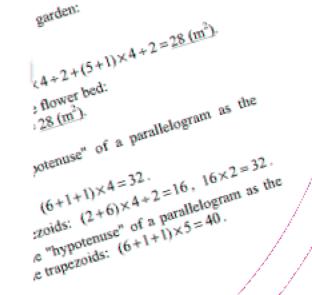
5

-2 = 93.

-5 = 37.2

ents consider the total of five times to be five times of 98 (average):

$$88 \times 4 = 352$$
, $98 \times 5 = 490$, $490 - 352 = 138$.



adding the

Finding of fourth and

Error Analys:
• (88+98) ÷

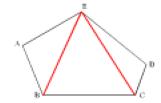
· (88+98)÷

. (00+30)

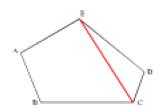
· Some stud

	Prob.	Problem	Ave. of correct answer (%)			Errors	
1	No.	Fronten	2012	2013	2014	(Rate to the whole errors)	
	(21)	A figure surrounded with five straight lines is called a pentagon. How many degrees is the sum of interior angles of the following pentagon ABCDE?	62	67	67	1. 900° (18%) 2. 720° (7%) 3. 520° (7%) 4. others (27%) 5. no answer (41%)	

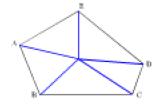
· Finding the sum of a polygonal interior angles based on "the sum of a triangle's interior angles is 180°."
Expected Thinking



 $180 \times 3 = 540$



360 + 180 = 540



 $180 \times 5 - 360 = 540$

• Sum of interior angles of the n-gon is $180 \times (n-2)$ in general. So, if n = 5, $180 \times (5-2) = 540$.

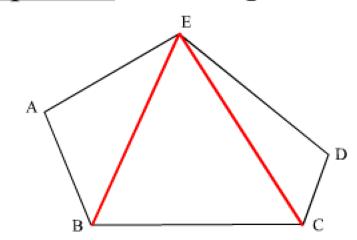
Error Analysis

- 180×5=900.
- 180 + 540 = 720, or 180 + 540 = 520 (miscalculation).

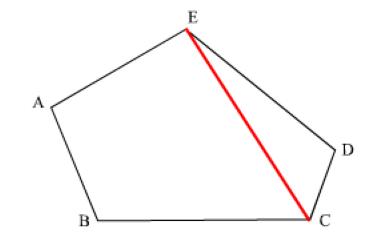
 $88 \times 4 \approx 352$

• Finding the sum of a polygonal interior angles based on "the sum of a triangle's interior angles is 180°."

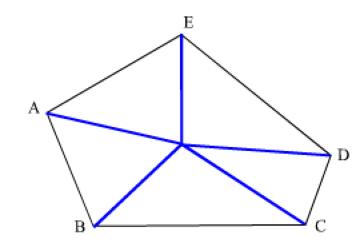
Expected Thinking



$$180 \times 3 = 540$$



$$360 + 180 = 540$$



$$180 \times 5 - 360 = 540$$

• Sum of interior angles of the n-gon is $180 \times (n-2)$ in general. So, if n = 5, $180 \times (5-2) = 540$.

- $180 \times 5 = 900$.
- 180 + 540 = 720, or 180 + 540 = 520 (miscalculation).



Grade

Problem of 6th Grade

Prob.	Problem	Ave. of	correct ans	Errors	
No.	Frobleiii	2012	2013	2014	(Rate to the whole errors)
(20)	T-shirt of 1200 yen has become to 960 yen at the bargain sale. What percent discounts?	40	45	44	1. 80 % (50%) 2. 1.25% (9%) 3. 125% (7%) 4. 5% (5%) 5. others (28%) 6. no answer (18%)

Intention of the Problem

· Understanding the proportion.

Expected Thinking

- 1280 960 = 240, $240 \div 1200 = 0.2$, $0.2 \times 100 = \underline{20}$ (%).
- $960 \div 1200 = 0.8$, 1 0.8 = 0.2, $0.2 \times 100 = 20$ (%).

- $960 \div 1200 = 0.8$, $0.8 \times 100 = 80$.
- $1200 \div 960 = 1.25$; $1.25 \times 100 = 125$.
- 1200 960 = 240, $1200 \div 240 = 5$.

- The efforts of preparing, implementing, and analyzing the regional assessment test by teachers groups contribute to their professional development, especially in terms of mathematical content knowledge study "kyozai-kenkyu."
- Furthermore, such a test is also effective for improvement of teaching in the classroom.

- The teachers groups have difficulty for "the development of new problems."
- If an international database of math problems is developed, it is hoped that it helps the solution to such difficulties.



Japanese Elementary School Students Math Performance: A Case of Tottori Prefecture regional "Math Diagnostic Test"

Tatsuya Mizoguchi Tottori University, Japan

Yusuke Shinno Osaka Kyoiku University, Japan



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